SCHOOL OF CIVIL ENGINEERING

INDIANA DEPARTMENT OF HIGHWAYS

JOINT HIGHWAY RESEARCH PROJECT

JHRP 86/12

INFORMATIONAL REPORT

A GENERAL METHOD FOR THREE DIMENSIONAL SLOPE STABILITY ANALYSIS

Jose E. Thomaz

PURDUE UNIVERSITY

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TO:	H. L. Michael, Director Joint Highway Research Project	August 27, 1986
		Project: C-36-36N
FROM:	C. W. Lovell Research Engineer	File: 6-14-14

The attached report is the first describing a new 3-D slope stability analysis. Programmed for microcomputers, it has the code name "3D PC STABL."

Several years ago Professor Lovell initiated consideration of 3-dimensional slope stability analysis following successful research on 2-dimensional analysis. In 1984, Mr. Thomaz was employed to handle the record keeping and sales of the several computer programs developed on 2-dimensional analysis of slope stability and paid from receipts of these sales. He also developed an interest in the 3-dimensional problem and following two years of work has produced the attached report which he also used in partial fulfillment of the requirements for the MSCE degree.

During the early years of slope stability research in the JHRP, a project had been proposed on 3-D analysis and a project and file number assigned. Professor Kovacs was also involved in this activity then but when he resigned from the University the proposal was not pursued. For record purposes, those project and file numbers have been utilized for this research report. As the sales activities of 2-dimensional slope stability analysis programs were used to employ Mr. Thomaz and required the time and effort for which he was paid, the effort by Mr. Thomaz on the 3-D research, which resulted in this report, was from his dedication to the subject and the desire to obtain an advanced degree. The research project, hence, was not approved as a JHRP project, not performed with JHRP funds, but rather as a contribution by Mr. Thomaz of his advanced degree studies for the Masters degree. The result is the attached Informational Report.

Although the research reported here is an initial development of 3-dimensional slope stability analysis, many topics need further development and are noted in the conclusions. Nevertheless, the computer analysis program developed will be useful to IDOH in practical problems where the slope instability is, or may be expected to be, strongly 3-dimensional. Since nearly all slope failures are three dimensional, the analysis has great potential utility. In all initial uses, it should be run in parallel with the 2-D analysis (PC STABL4 or PC STABL5) to determine how the factor of safety is being changed.

The program listing and examples are 167 pages in length and are not included in this report as normally distributed.

Respectfully submitted,

C. W. Lovell

C. W. Lovell
Research Engineer

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Informational Report

A GENERAL METHOD FOR THREE DIMENSIONAL SLOPE STABILITY ANALYSIS

by

Jose E. Thomaz
Graduate Instructor in Research

Joint Highway Research Project

Project No.: C-36-36N

File No.: 6-14-14

This non-sponsored research project was performed by the author under the direction of Professor C. W. Lovell, School of Civil Engineering, Purdue University, in partial fulfillment of the requirements for the M.S.C.E. degree.

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August 27, 1986

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The author is greatly indebted to his advisor, Professor C. W. Lovell, for his continuous support, useful advice and inspiration provided during the research period, and for his friendship on all occasions.

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LIST OF SYMBOLS AND ABBREVIATIONS

Abbreviations

2D - Two Dimensional

3D - Three Dimensional

Special Symbols

A prime (" ′ ") indicates that the variable is in terms of effective stress.

(Delta) indicates a change or variation in the variable.

(Sigma) is used as summation symbol.

Symbols

Ab - Area of the base of column

c - Strength intercept of the soils

F - Factor of Safety

FS - Factor of Safety

Fi - Forces in general

hc,q - height of point of application of force

hi,j - height of layer i or j

Ko - Coefficient of lateral pressure

Q - Resultant of all side and shear forces acting on a slice (column)

N - Normal force acting on that side or on the base of the column

r - radius of circle

Ru or ru - porepressure parameter

Ri,j - Resulting force on side i , j

Rext - Resulting normal external force on side of columns

Sext - Resulting shear force on external side of columns

u - Pore water pressure

W - Weight of a mass

X - Coordinate of X cartesian axis

Y - Coordinate of Y cartesian axis

Z - Coordinate of Z cartesian axis

Greek Alphabet

α ij - Inclination of base of column in direction ij

γ - Unit Weight of a soil

θ - Parallel inclination of all forces

ϕ i - Shear strength angle of a soil i

ABSTRACT

This work presents a methodology to generate random three dimensional sliding surfaces and evaluate their factor of safety. The method employs random numbers to create a surface within boundaries defined by the user. Angular restrictions are imposed in order to keep the surface kinematically compatible. The soil is divided in columns, and the forces on the sides are calculated. The equilibrium of forces and moment is satisfied. The methodology was implemented in a program that runs on IBM microcomputers. Graphical processors were also developed to help checking the input of data and the output of the results. The resulting system is called 3D-PCSTABL. Some cases were run comparing the effects of the inclination of the slope and of the strength parameters on the shape of the critical surfaces and on the ratio between the 3D factor of safety and the traditional 2D factor of safety. It has been found that the steeper the slope, the lower the ratio between 3D and 2D factors of safety for medium and highly cohesive soils. For cohesionless soils the behavior is the opposite, with the ratio increasing for steeper slopes. In cohesionless soils, the critical three dimensional surfaces were found to be deeper than the most critical two dimensional surfaces, for all slopes. The higher the cohesion of the soil, the flatter the slope, the better the agreement between the two and three dimensional most-critical surfaces. The three dimensional shapes of the surfaces were found also to be related to both the strength parameters and the inclination of the slope.

Cohesionless soils in general showed narrower, blocklike surfaces, with a tendency to widen as the slope flattens. Cohesive soils on the contrary showed a tendency to have wider, ellipsoidal or spherical surfaces.

I - INTRODUCTION

In the last 20 years, the area of slope stability has received a lot of research attention. Most diverse approaches have been developed in the search of a better, more realistic, representation of the physical phenomena involved in a landslide. These approaches include limit equilibrium analysis, finite element analysis and, more recently, variational calculus techniques. All of these ways for approaching a slope stability analysis have their strong and weak points .

The purpose of this work was to develop a program with features to let the three dimensional analysis of slopes be performed with relative simplicity . For this purpose, a routine for the generation of three dimensional surfaces was developed. This routine will let the program perform an automatic search for the critical 3D-surface, inside a specific region of the slope. A general method for the analysis of the equilibrium equations was also incorporated, and both forces and moment equilibrium are satisfied. The program code is named 3D-PCSTABL.

The approach used in the development of this work is that of limiting equilibrium. This kept the selection of parameters for soil reprèsentation simple, as well as retaining the flexiblility

to handle diverse geometric profiles and ground water conditions.

Although the program presented here is powerful and flexible, it still suffers some limitations , characteristic to the limit equilibrium approach . These limitations include the assumption that the soil mass slides as a solid body, without any forced compatibility of stress-strain relations; the assumption that the factor of the safety is the same all along the surface, as well as inside the sliding soil mass; and consequently, the usual ignoring of progressive failure phenomena.

On the other side, the program presents an efficient approach to perform studies of stability when the geometry of the problem is not conveniently represented two dimensionally.

The program uses three dimensional randomly generated surfaces to search for the position and shape of the most critical failure surface within a selected zone of the soil mass. Each surface has its factor of safety evaluated and the 10 most critical ones are displayed after a pre-determined number of them have been generated. If the surfaces show a well defined pattern, the region for the search can be re-defined, until the most critical zone of the slope has been identified with acceptable accuracy. This procedure follows the approach used in the program STABL (Siegel,1975), developed for two dimensional analysis of slopes at Purdue University.

By means of a flexible geometry representation , the program can handle surfaces of weakness, as discontinuities and bedding planes.

To help the visualization of the input geometry and resulting critical region, graphical processors have been developed. These routines display the three dimensional soil profiles on the screen and allow both entered data and final results to be checked. The plots can be rotated about the three reference axes, in any combination of angles. The image can also be dumped to a dot matrix printer , at the user's command.

The program, 3D-PCSTABL, has been developed to run on IBM-PC microcomputers and is written in Microsoft FORTRAN 77 , version 3.2 , for the sake of portability . The minimum memory required is 256 Kb . Nevertheless ,this minimum limits the flexibility of the routine, and a larger memory (640 Kb) will allow more sophisticated analyses to be performed.

If the minimum amount of memory is to be used , a numerical co-processor (Intel 8087) is necessary. This happens because the compiled code of the program is more compact when the coprocessor is present.

The graphical processors are also written in Microsoft FORTRAN version 3.2 , and the graphical commands are provided by

the GRAFMATIC scientific/engineering graphical library , written specifically to the IBM-PC .

As a consequence , the program , with exception of the graphic commands, is portable to any other system that supports FORTRAN language , with no or minor modifications.

II - A REVIEW OF LIMIT EQUILIBRIUM METHODS FOR THREE DIMENSIONAL ANALYSIS OF SLOPES

Very few methods have been developed for three dimensional slope stability analysis , and most of them are restricted to simple soil geometries and water conditions, being therefore , not useful for many practical purposes.

Sherard et al. (1963) introduced the concept of a weighted average factor of safety , resulting from the analysis of several two dimensional sections across the slope (FIGURE II.1). Each of these sections would have an area A(i) and a two dimensional factor of safety F(i). Thus the final averaged factor of safety FS would be :

$$FS = \frac{\sum A(i)\ F(i)}{\sum A(i)} \qquad \text{(Eq. 2.1)}$$

This approach usually gives factors of safety lower than the next method , which consists of the inclusion of end effects on a traditional two dimensional analysis.

Baligh and Azzouz (1975) extended the concept of the two dimensional circular arc shear failure to three dimensional

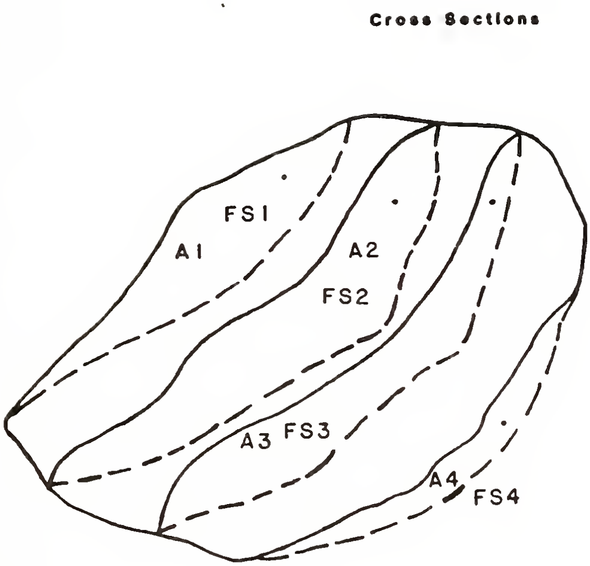


FIGURE II.1 - Two Dimensional Cross Sections on a Three Dimensional Slope

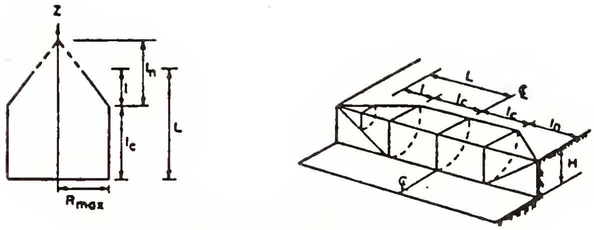
problems by introducing either ellipsoids or cones at the extremities of a cylinder with finite length 2L (FIGURE II.2). They studied the effects of the surface ends on cohesive slopes and concluded that end effects increase the factors of safety obtained from two dimensional analysis . For shallow surfaces, with a ratio between the length along the slope and the depth of the failure greater than eight , the difference is less than 5% and can be neglected. On the other hand , if the ratio is less then four , the increase can be of the order of 30% or higher and a three dimensional analysis is then relevant.

Slopes having variable soil properties and or variable cross sections may be subjected to larger influence of end effects than homogeneous uniform ones .

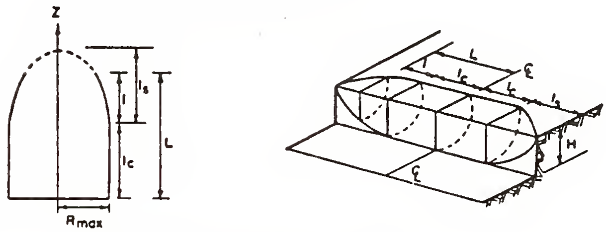
Hovland (1977) suggested a method which expanded the ordinary method of slices to three dimensions. Instead of slices, columns were used and for simplification purposes , the inter-column forces were neglected. The factor of safety was defined as the ratio between the total available resistance along the sliding surface and the mobilized stress along it.

Another assumption was that there was motion only in one direction . The equilibrium of the system was calculated for this direction.

The results of Hovland's studies suggested that the shape of



(a) Cone Attached to a Cylinder



(b) Ellipsoid Attached to a Cylinder

FIGURE II.2 - Geometries of Failure Surfaces and their Plan Views; (After Chen,1981)

the three dimensional critical surface and ratio between the three dimensional and two dimensional factors of safety are sensitive to the soil strength parameters c and ϕ . A general conclusion was that for cohesive soils , the critical surface tends to be long and the two dimensional approach is usually a reasonable approximation . Yet, for cohesionless soils , the surfaces are narrower and a three dimensional analysis may be more representative.

Chen (1981) developed a very comprehensive study of the three dimensional effects on slope stability for a large variety of soil parameters. He suggested methods for the analysis of three dimensional block surfaces (FIGURE II.3) as well as for rotational surfaces (FIGURE II.4) . The methods expanded the concepts introduced by Hovland and included the inter-column forces in the analysis.

His assumptions included symmetry of the sliding surface in the third dimension , soil strata lateral continuity , water surface far below the surface, and a unique factor of safety for the whole failure surface .

The block analysis divides the sliding mass in three parts : a passive block , a central block and an active block. The system satisfies force equilibrium in the vertical and horizontal directions. Chen's study of translational slides concluded that

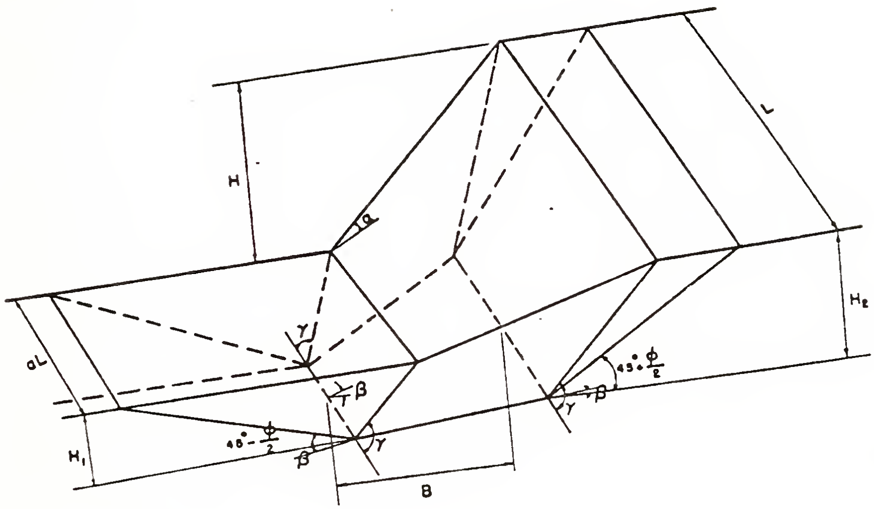


FIGURE II.3 - Three Dimensional Block Type Failure Surface; (After Chen,1981)

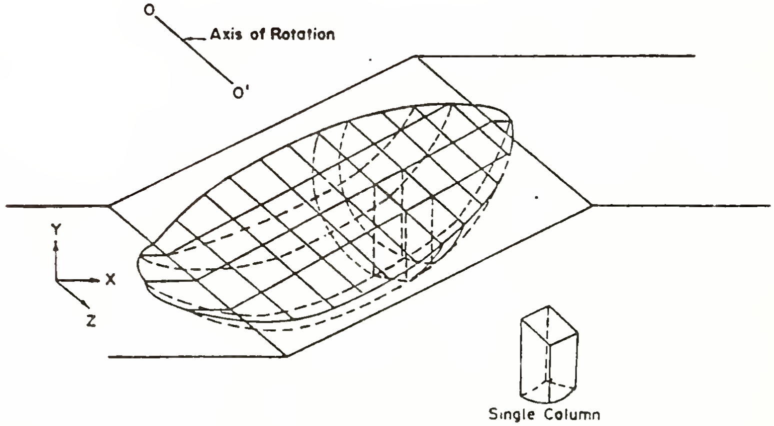


FIGURE II.4 - Rotational Spoon Shaped Failure Surface;
(After Chen,1981)

even though 3D factors of safety are usually larger than 2D factors of safety , wedge shaped failure surfaces may generate ratios F3D/ F2D less than unity. He also found that 3D effects are more significant on cohesive soils than cohesionless soils.

The analysis of rotational failure mechanism combines cylindrical surfaces with semi-ellipsoidal or conical extremities. The intercolumn forces are taken as parallel to the base of the column and are a function of their position. The inclination of interslice forces are the same throughout the whole failure mass. Both force and moment equilibrium are satisfied for each column as well as for the total mass. Chen's analysis concluded that the three dimensional effects are more significant at smaller lengths of the failure mass and for highly cohesive soils than for cohesionless soils, as already discussed for the block analysis .

The program presented in this work expands the concepts presented by Chen to a surface of any shape . A detailed description of the concepts and hypothesis involved will be presented in Chapter Five.

III - DESCRIPTION OF THE GEOMETRY OF THE PROBLEM

The main obstacle to the adoption of three dimensional methods on a daily work basis is the difficulty in establishing the shape and position of the slipping surfaces to be analyzed. Geotechnical engineers are already accustomed to two dimensional slope analysis , and most engineers can estimate intuitively different shapes for candidate surfaces, during the analysis of a given two dimensional homogeneous soil profile. But if different layers of soils are included, each with different strength properties , and if complex ground water conditions are introduced , this task can be progressively more difficult .

Under such conditions, most engineers start losing the sensibility for determining where the critical region of the slope is , and trial and error numerical procedures start to be necessary.

Now , if we assume that this already complex two dimensional profile , is nothing but a simplification of the even more complicated and real three dimensional in situ conditions, it becomes evident why the three dimensional approach has not become a popular method for slope stability analysis.

This chapter will describe how a practical and efficient

routine for generation of three dimensional surfaces has been developed . The use of this routine, with an appropriate method of analysis will , hopefully , bring the three dimensional slope stability analysis to the same currency of use as for two dimensional methods . Thus engineers will be able to perform a more representative analysis , in situations where the in situ conditions can not be appropriately reproduced in a common two dimensional approach.

III.1 - Geometry Representation

The first problem that had to be overcome was to find an efficient way of representing the soil profile . This method had to be capable of representing diverse profiles and different ground water conditions , and more importantly , it should contribute to efficiency in the process of searching for the critical surface.

The system for representing the geometry of the problem will consist of a three dimensional cartesian system. The engineer will define three orthogonal axes , "X" , "Y" and "Z" . The horizontal plane will be defined by axes "X" and "Y". The axis "Z" will represent the elevations of the layers and ground profiles. During the determination of this coordinate system , the engineer should keep in mind that the direction of the slides

being investigated is along the axis "Y". FIGURE (III.1) shows the representation described above .

The next step consists of defining the points (coordinates) that will represent the geometry. The way these coordinates are to be defined can be seen in FIGURE (III.2). The user defines a grid , consisting of cross sections parallel to the "X" and to the "Y" axis. The spaces between these cross sections do not need to be the same. Nevertheless , it is highly recommended that they do not present large variations .

The resulting mesh can be seen as a series of axes in the "X" direction and another series in the "Y" direction . The intersections are the points that will define the geometry of the problem.

These points can be perceived as locations of boreholes . For each of them , the elevations of the ground surface will be defined , as well as the elevations of the different soil layers and piezometric levels .

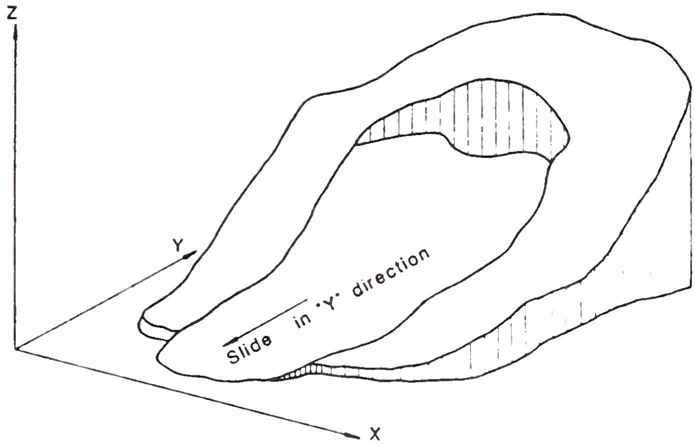


FIGURE III.1 - System of Coordinates

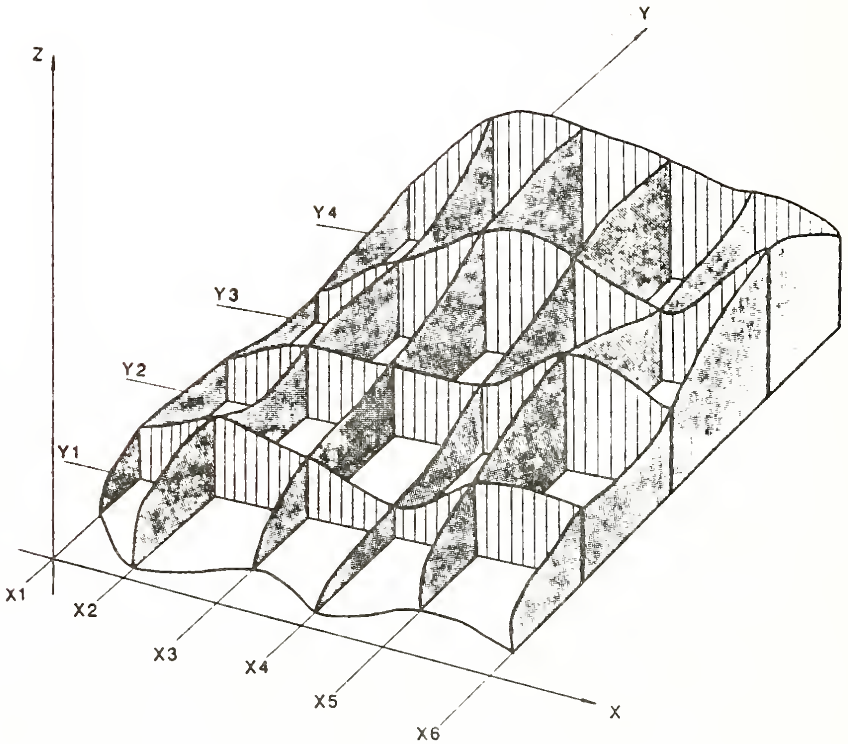


FIGURE III.2 - Grid of Cross Sections

III.2 - Soil Boundaries

The soil boundaries will be defined by specifying the elevations of the different soil layers at each of the nodes of the grid. The resulting profile will be approximated by line segments connecting the layers. The selection of the position of the cross sections should be made in such a way that no important feature of the subsoil is incorrectly represented. To help in this selection, the developed program allows the geometry to be checked , by displaying the plotted layers three dimensionally on the screen .

III.3 - Piezometric Surfaces

The piezometric surfaces involved in the analysis will be defined in the same way as the soil layers . For each point of the mesh , the user will define an elevation corresponding to each of the piezometric surfaces being described.

The piezometric surfaces may lie anywhere within the region being defined. They can be above the ground surface or coincide with it at different points. If more than one piezometric surface is defined, and one of them is going to represent a water body, such as a lake or river, it should be assigned to the first layer

of the soil. The program will check if the first piezometric surface lies above the ground , and wherever it does , it will add the weight of water to the calculations of the total weight of the columns. If the piezometric surface above the ground surface is not the one assigned to the top layer, the water will only be considered for pore pressure calculation purposes.

III.4 - Definition of Searching Boundaries

In order to optimize the surface searching process , appropriate boundaries must be defined . The choice of appropriate boundaries is important to reduce the amount of time involved in the search, by avoiding the generation of "useless" surfaces . "Useless" surfaces are those with little or no probability of being critical (as surfaces passing through sound rock , for instance). Some engineering judgement is thus required, when defining the region to be searched .

It should be noted that even if the boundaries are very wide, the resulting critical region will still be the same . Nevertheless, the amount of time necessary to arrive at this final region will be much larger , since a higher percentage of the surfaces generated will probably lie outside of the critical region of the slope.

The definition of the boundaries contains three different steps:

1) Definition of the main sliding axis

2) Definition of the regions within which the surfaces are supposed to start and to end , on the ground surface.

3) Definition of the region within the soil mass that the program is supposed to search.

These steps will be discussed in the following sections .

III.4.1 - Definition of the Main Sliding Axis

The program assumes the existence of an axis of sliding , which is to be defined parallel to the "Y" axis (FIGURE III.3). The whole surface generation process will be dependent on this "main" axis. This axis will actually be one of the cross sections already defined according to Section III.1 . Further details about the function of this main axis will be given when the surface generation process is described (Section IV.2.1).

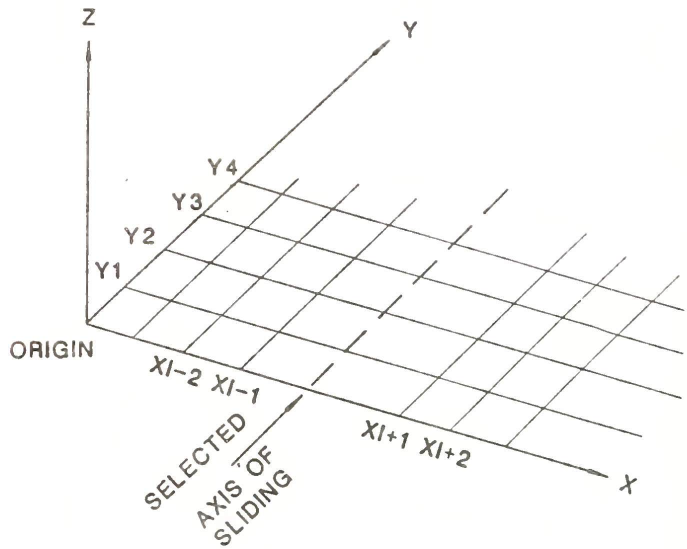


FIGURE III.3 - Main Axis of Sliding

III.4.2 - Definition of the Regions Within Which the Surfaces Should Start and Should End on the Ground Surface.

Boundary restrictions must be defined in both "X" and "Y" directions. The best way to understand how these boundaries are to be established is to imagine viewing the slope from above. The cross sections previously defined now appear as lines, perpendicular to each other (FIGURE III.4) .

III.4.2.1 - Defining the Limits in The "Y" Direction

The limits in the "Y" directions will be defined on the main axis of sliding. To establish the region within which the beginning points of the generated surfaces are expect to lie, define a minimum value of "Y" and a "delta Y" (FIGURE III.5). The procedure should be repeated for the region where the surfaces are expected to end (FIGURE III.5).

III.4.2.2 - Defining the Limits in the "X" Direction

The procedure to define the limits in the "X" direction is basically the same , but now these regions must be defined for each of the "X" direction cross sections contained within the

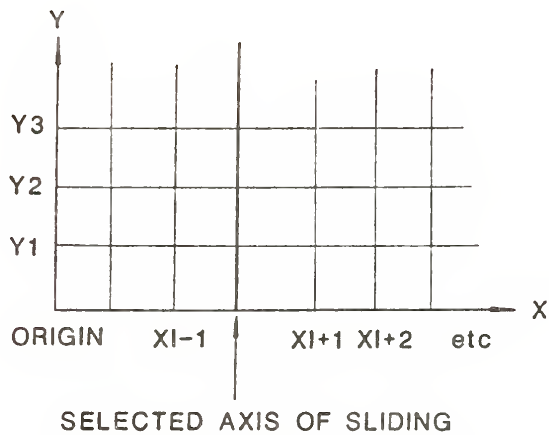


FIGURE III.4 - Top View of Grid

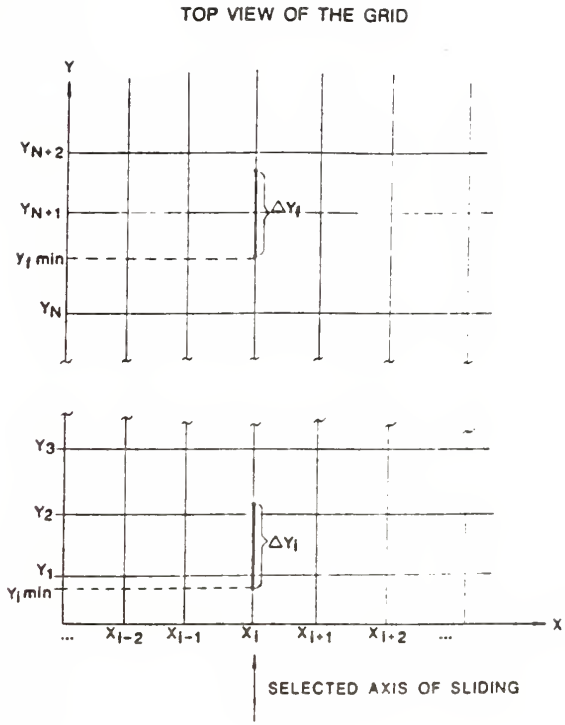


FIGURE III.5 - Searching Boundaries in the "Y" Direction

limiting regions defined previously on the main sliding axis for the "Y" direction (FIGURE III.6).

III.4.3 - Definition of the Region Whithin the Soil Mass that the Program Will Search.

When the preceding steps have been acomplished , the last part of the boundary definition process is to define the region inside the soil mass which is going to be searched.

The necessity of these boundaries will become evident when the routine for generation of surfaces is discussed . If we consider that the process of generating a three dimensional surface typically requires times between 40 and 70 longer than those for the generation of a two dimensional surface (not considering the time for analysing them) , it becomes evident why the program must avoid generating surfaces out of the region of primary interest.

The way to define the boundaries for the search inside the soil mass (in the "Z" direction) is very similar to that formerly described . For each of the points that were previously used to define the soil profile, a minimum depth "zmin" and an interval "delta z" will be defined . These will limit the region within which the surface is allowed to pass (FIGURE III.7).

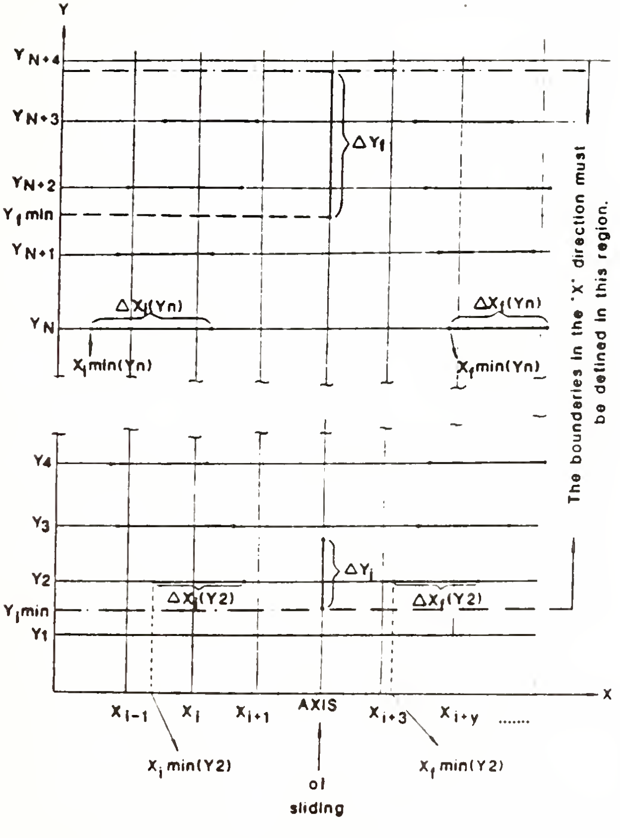


FIGURE III.6 - Searching Boundaries in the "X" Direction

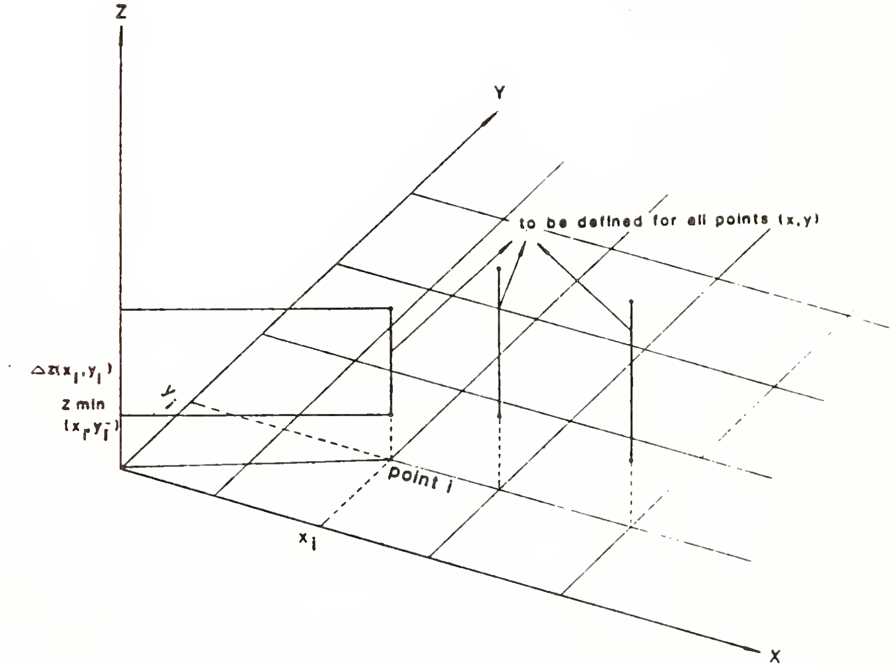


FIGURE III.7 - Searching Boundaries in the "Z" Direction

These restrictions are useful in restricting the search to weak layers , in avoiding rock formations , or in progressively optimizing the region to be searched , based on the results of former runs of the program.

The routine for surfaces generation contains automatic checks and numerical bias to control the shapes of the surfaces , but the imposed search limit boundary restrictions always have precedence over any other methodology used in the process .

IV - A GENERAL ROUTINE FOR RANDOM GENERATION OF THREE DIMENSIONAL SURFACES

IV.1 - Methodology of the Random Generation

The idea of using random numbers to generate surfaces has already been used in the well known program for two dimensional analysis of slopes called STABL . This program , developed at Purdue University (Siegel , 1975) , contains three different routines for random generation of surfaces . The program generates surfaces with circular , block like or irregular shapes.

The methodology used in the routine to be presented here differs a little from the one used in STABL. The main difference lies in the fact that STABL generates surfaces at random and periodically checks whether the surface is meeting the required boundary conditions or not. If not , the program tries to modify the surface. If it still fails to pass the check , it abandons the surface and start generating another. The routine presented here will always generate surfaces that meet the imposed boundary conditions , in all steps of the process.

The STABL approach , under certain very restrictive boundary conditions , forces the program to generate up to three or four times the number of surfaces required , in order that a sufficient number of surfaces meet the conditions. In other words, if 50 surfaces are required , the program may have to generate up to 200 surfaces , from which 150 will be abandoned somewhere in the process of generation. It is true that the above case is an extreme one , and the average number of surfaces abandoned should be expected to be of the same order as those required, i.e. for each two surfaces generated, one is abandoned.

The most efficient routine in STABL so far as computing time is concerned, is the BLOCK routine. By specifying boxes , within which the vertices of the surfaces are forced to lie , the number of surfaces abandoned is decreased to practically zero.

The approach used in this routine for generation of three dimensional surfaces, resembles somewhat the concepts involved in using the boxes . The difference lies in the fact that vertical lines , instead of boxes , are used .

IV.2 - The Three Steps of the Surface Generation

The process of random generation of each three dimensional surface has three independent steps:

1) Generation of the axial two dimensional main cross section.

2) Generation of the intersection between the sliding surface and the ground surface.

3) Generation of the body of the sliding surface.

These three steps are executed by different routines , but the principle that lies behind them is the same. Random numbers are used to select the position of the points that will compose the sliding surface . Bias is introduced to keep the surface from taking inconsistent shapes. A more detailed discussion about the admissibility of the shapes generated will follow the presentation of the numerical procedures.

IV.2.1 - The Generation of the Main Axial Cross Section

The first step in the generation of a three dimensional surface is the generation of the main two dimensional cross section of the surface. In Section III.4 , it was stated that a main axis of sliding should be defined. This axis has a very important place in the generation of the surface .

The routine will first generate a two dimensional surface , which will lie on the main axis of sliding. The initial point of the shear surface will lie on the ground and its coordinates will be given by :

$$X(1) = X(\text{main axis}) \qquad Y(1) = Yimin + R * deltaYimin \qquad \text{(Eq. 4.1)}$$

where Yimin is the initial point, and deltaYimin is the length of the interval specified for the begining of the surfaces (FIGURE IV.1), and R is a random number. The same procedure calculates where the surface will terminate (FIGURE IV.1).

The rest of the two dimensional surface is calculated using the searching intervals defined according to Section III.4.3 . The coordinates of the points will be calculated as

$$\begin{aligned} X(i) &= X(\text{main axis}) \\ Y(i) &= Y(\text{cross section where point i lies}) \qquad \text{(Eq. 4.2)} \\ Z(i) &= Zmin(X(i),Y(i)) + R * DeltaZ(X(i),Y(i)) \end{aligned}$$

where Zmin is the minimum depth and DeltaZ is the interval where the surface may pass through at that location (FIGURE IV.2) , as defined in Section III.4.3 .

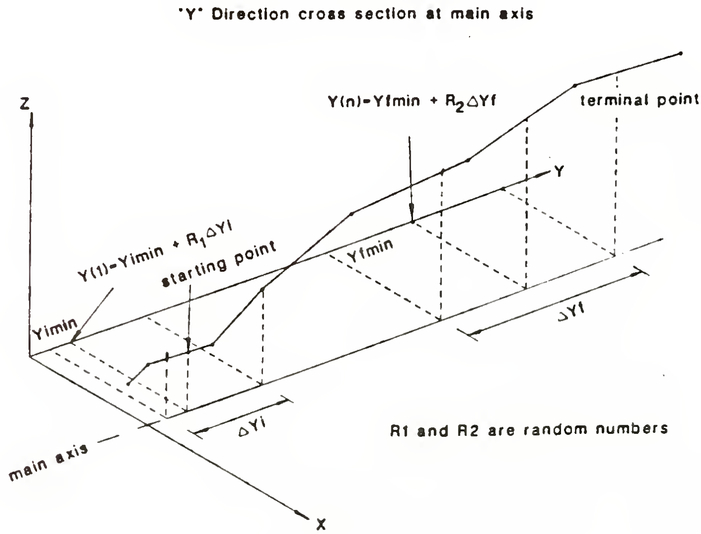
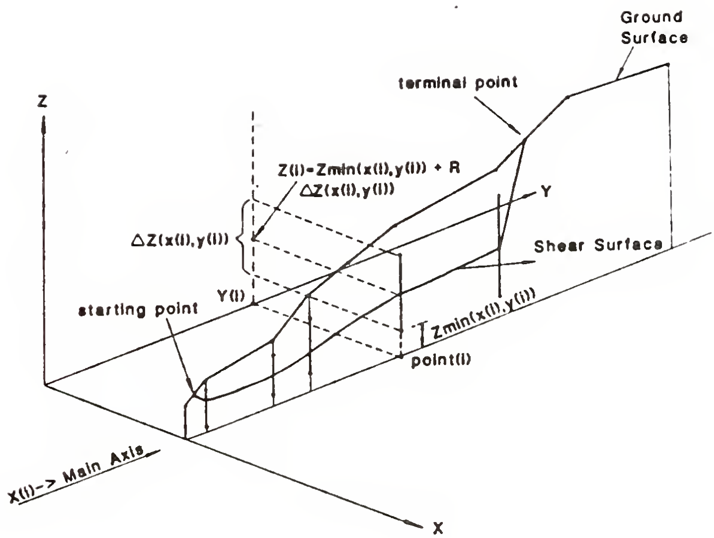


FIGURE IV.1 - Generated Extremities of Surface in "Y" Direction



R is a random number

FIGURE IV.2 - Axial Two Dimensional Surface

Some bias has been introduced in the automatic control of the angles between the line segments that form the surface , to mantain a kinematically acceptable shape . The first line segment is calculated using two biased angle limits , as suggested by Siegel (1975) . The inclination of this segment is defined as

$$\text{theta} = \text{alpha2} + (\text{alpha1}-\text{alpha2}) * R^2 \qquad \text{(Eq. 4.3)}$$

were R is a random number , alpha2 , the clockwise direction limit , is -45^o , and alpha1 , the counterclockwise direction limit , has been set as 5^o less than the inclination of the ground at the initiation point (FIGURE IV.3) . This expression was suggested by Siegel (1975) and is appropriate for the current situation. After the line segment angle has been calculated , the program checks whether it will pass through the region defined to contain the surface . If it does, the angle is kept and the second segment is calculated . If it passes above or below the region , the program moves the point to be at the upper or bottom limits of the defined interval , and recalculates the angle theta accordingly (FIGURE IV.4).

From the second to the last line segment , the process of generation will be a little different . A coordinate Z(i) , for the point to form the i^{th} segment , will be calculated according to equation 4.2 , and the angle between the line segment just

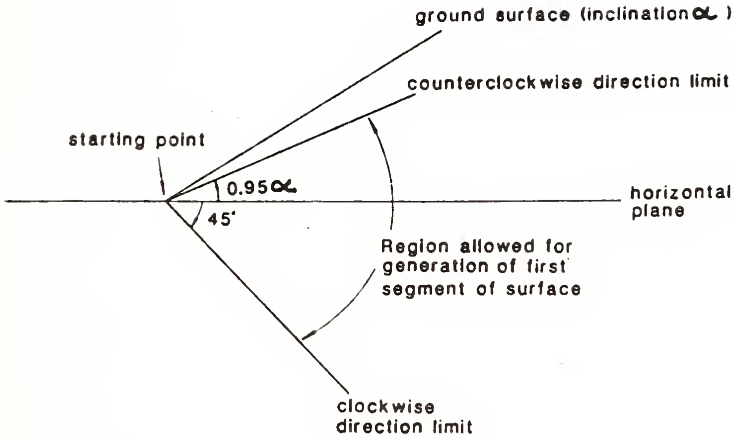


FIGURE IV.3 - Angle Generation for First Line Segment of Axial 2D Surface

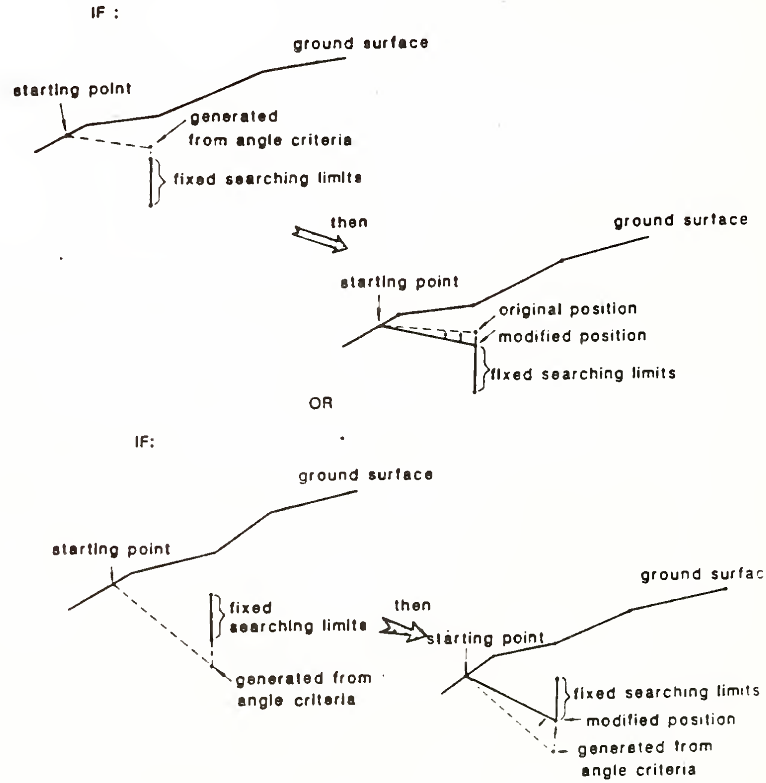


FIGURE IV.4 - Adjustment of Points According to Limits

formed and the previous one will be checked according to the expression

(delta theta = theta of segment "i" minus theta of segment "i-1")

$$\text{delta theta} = \text{dtheta2} + (\text{dtheta1} - \text{dtheta2}) * R^{R} \qquad \text{(Eq. 4.4)}$$

where R is a random number ,dtheta2 , the clockwise direction limit is $-(R^2 * 45^o)$ and dtheta1, the counterclockwise direction limit is 45^o (FIGURE IV.5).

From the second line segment to the last one , if the coordinate Z(i) given by expression 4.2 generates an angle compatible with the limits defined above, the coordinate is kept. On the other hand, if the angle between the line segments is larger than the limits, the coordinate is moved up or down ,accordingly , to make the restrictions valid (FIGURE IV.6) .

Lastly , it should be noted that , if at any moment the boundary intervals are such that it will not be possible to keep the angle restrictions, the specified boundary conditions have priority , and the angular restrictions are relaxed for that point (FIGURE IV.7) .

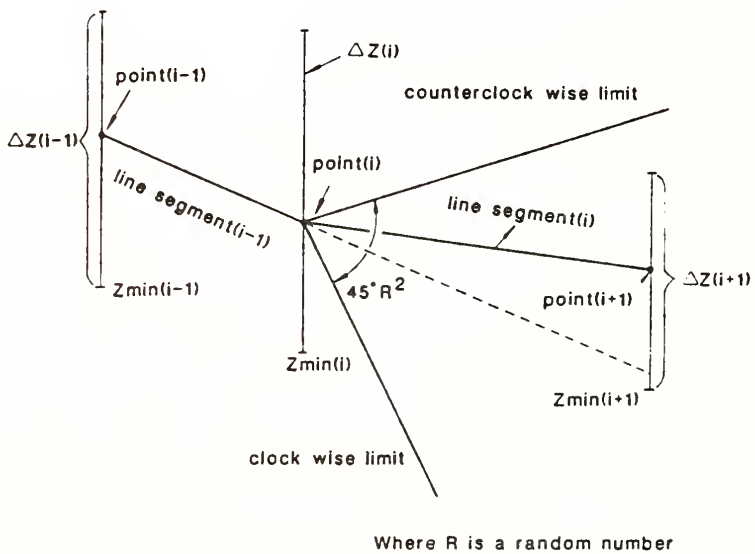
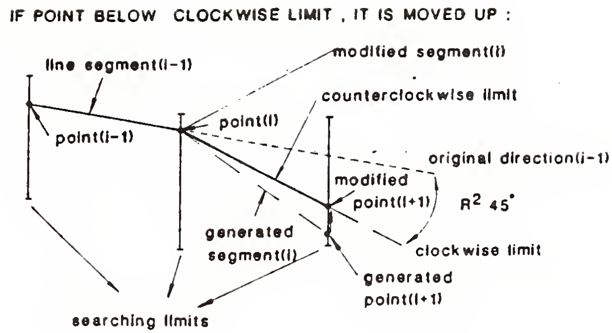


FIGURE IV.5 - Angle Limits Between Line Segments



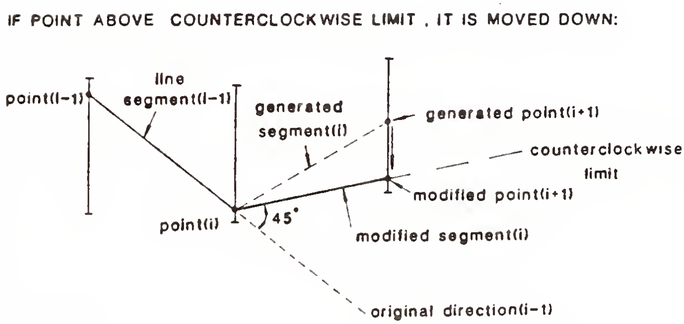


FIGURE IV.6 - Coordinate Adjustment Based on Angle Criteria

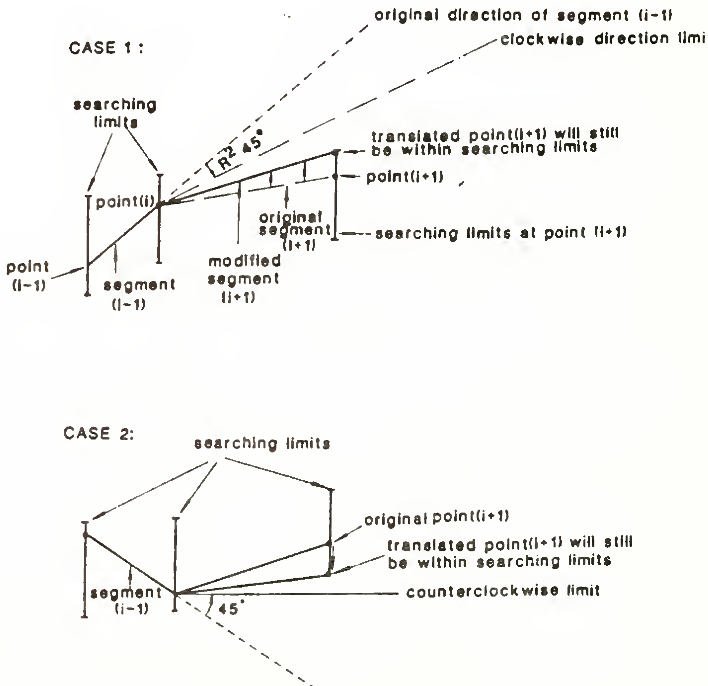


FIGURE IV.7 - Situations when Angle Criteria is Relaxed

IV.2.2 - The Generation of the Intersection Between the Sliding Surface and the Ground .

Once the program has generated the main axial cross section of the sliding surface, it generates the intersection between the shear surface and the ground . For this task the routine will use the intervals defined in Section III.4.2 .

The process of this generation is very similar to the one described in the previous section . This time , instead of searching intervals in the "Z" direction , the routine will use the boundary intervals defined in the "X" direction. These intervals will be the ones between the two points generated on the main axis , the starting and ending points of the surface in the Y direction, respectively (FIGURE IV.8).

At this point it is appropriate to introduce the idea of symmetry in the random generation. As it will be shown later , the hypotheses for the solution of the system of equations resulting from the three dimensional analysis include symmetry of the forces with respect to the main axis of sliding. Thus it is important that the surfaces be symmetric or , at least , not very far from symmetric.

The procedure that the routine follows is very general but , to allow the generation of symmetric surfaces, the process was

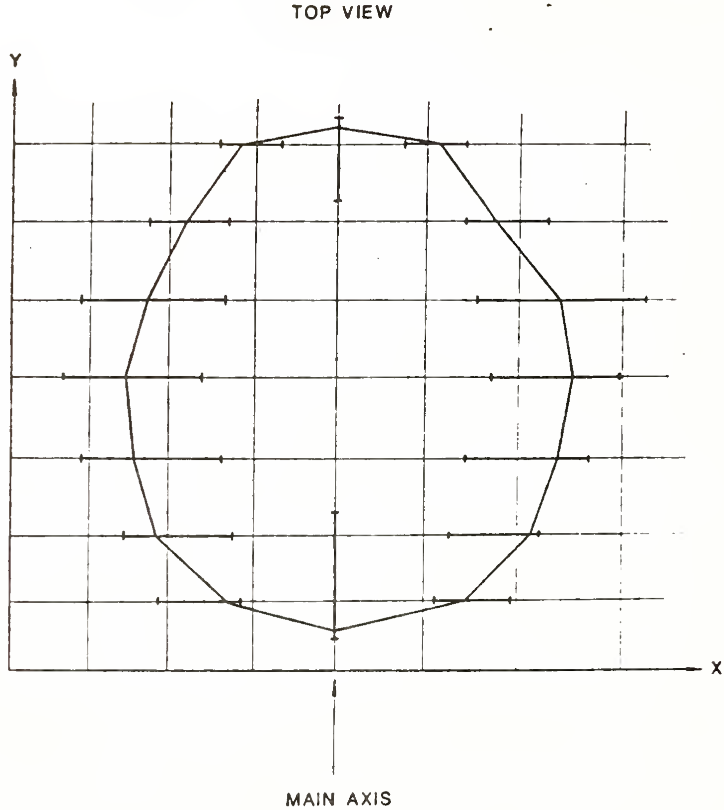


FIGURE IV.8 - Generation of Intersection Between Ground and Failure Surface

divided in two parts , both with identical steps. Instead of one random generator for the whole surface , there are three. One is for the main axis cross section , one is for the left side of the surface and another is for the right side . These last two generate the same series of numbers.

Random number generators make use of a seed number to start generating the series of numbers . If the same generator and same seed are used , the sequences will be the same . Thus by using this fact, and making sure that the boundary restrictions and searching intervals are symmetric , or at least not very different , the engineer can generate symmetrical , or almost symmetrical surfaces. In the future , when new methods are developed for non-symmetrical surfaces, different seeds can be used for the random generators , that will allow the generation of non-symmetrical surfaces , even for symmetrical boundary conditions.

The program takes the boundary intervals defined in the "X" direction and connects points randomly picked out on each one of them to form the line that defines the intersection between the shear surface and the ground surface. Each of the sides of the surface , with respect to the main axis of sliding , is generated separately. The coordinate X(i) of a point "i" is generated as shown in FIGURE IV.9 .

$$X(i) = X_{min} + R * \text{delta } X \qquad \text{(Eq. 4.5)}$$

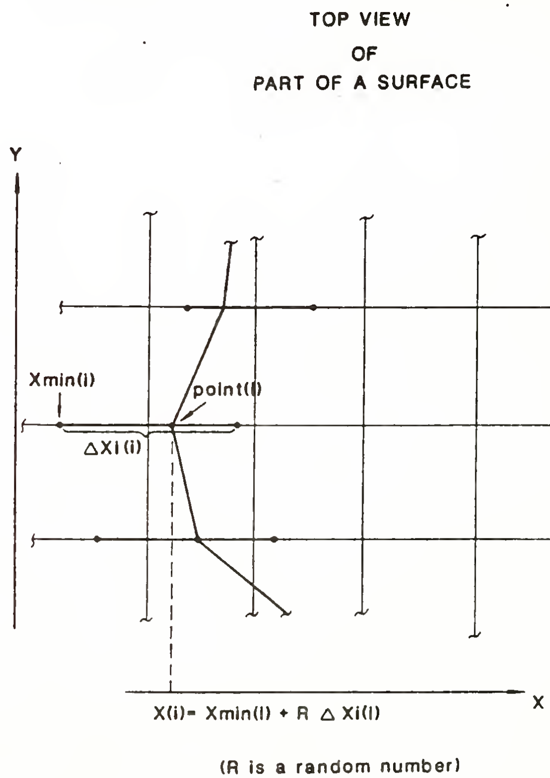


FIGURE IV.9 - Random Generation of the Coordinates of the Intersection Between the Ground and the Failure Surface

Also , Y(i) is the Y of the cross section where point i lies, Z(i) is the elevation of the ground at that set of coordinates (X(i), Y(i)), and it is calculated by the program. The variables Xmin and delta X have been defined in Section III.4.2.b.

When the points are generated , they are also subjected to restrictions applied to the angles between the line segments that connect them (FIGURE IV.10). These restrictions follow the same formulation presented in Section IV.2.1 , more specifically , equation 4.4 .

IV.2.3 - Generation of the Body of the Sliding Surface

The last step of the surface generation is the generation of what will actually be the shear surface. The process used here is basically the same one used thus far. But there is one small difference , with respect to the bias introduced in the control of the angles between the line segments .

The procedure of the routine for this part consists of the following steps :

a) Select one side of the surface.
b) Select the first cross section in the X direction, between the starting and ending points of the surface in the main axis of sliding.

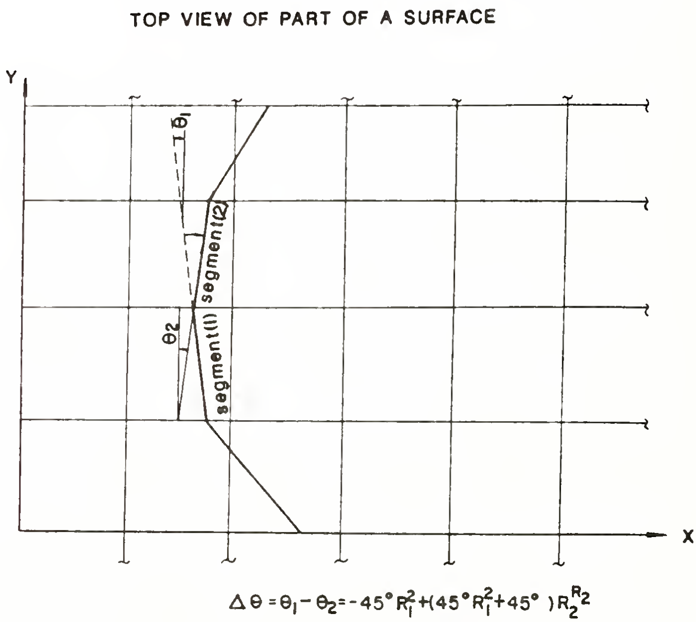


FIGURE IV.10 - Angle Restriction Criteria for Intersection Generation

c) For this section , the program will generate a shear surface in the X direction .

d) This surface will have its starting point at the point already generated at the main axis of sliding , for the corresponding "Y" coordinate of the cross section being analysed.

f) The ending point will be the point already calculated for the intersection between the surface and the ground .

g) The second point of the surface will be generated using equation 4.3 but now alfa2 is set to zero degrees. This means that for any cross section parallel to the X axis, the deepest point will always be at the main axis of sliding .

h) The points in between will have their elevations generated according to the usual formulation : Z(i) = Zmin + R * delta z ; were Zmin and delta z defined the searching interval for point "i" .

i) The program will also control the angles between the line segments connecting the points , and equation 4.4 will be used for this purposes .

j) The process is repeated for all cross sections between the starting and ending points in the main axis.

k) The process is repeated for the other side of the axis.

The important parts of the process can be seen in FIGURES IV.11 and IV.12 . Examples of three dimensional surfaces generated by the routine are presented in FIGURES IV.13 , IV.14 and IV.15 . These plots were generated in medium resolution by a small implementation of the routine , written in IBM BASIC . Non symmetrical surfaces were also generated for demonstration purposes .

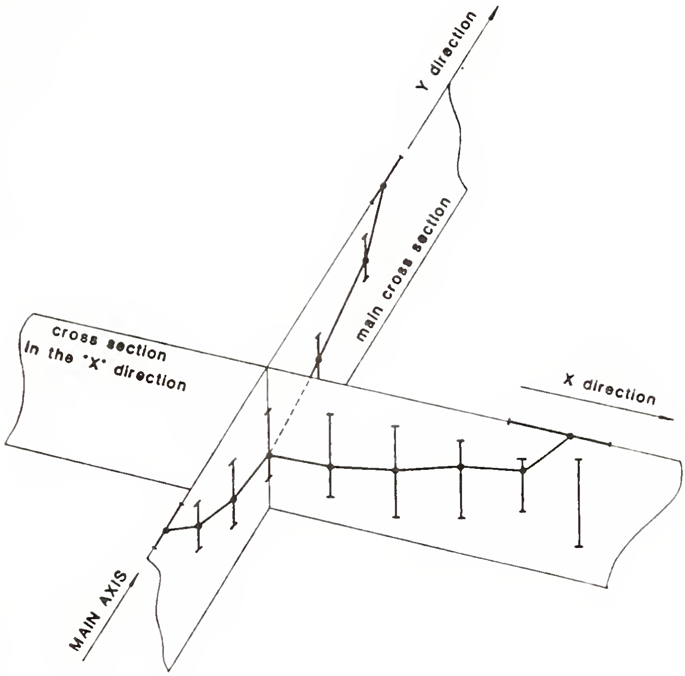


FIGURE IV.11 - The Process of Generation of a Three Dimensional Surface

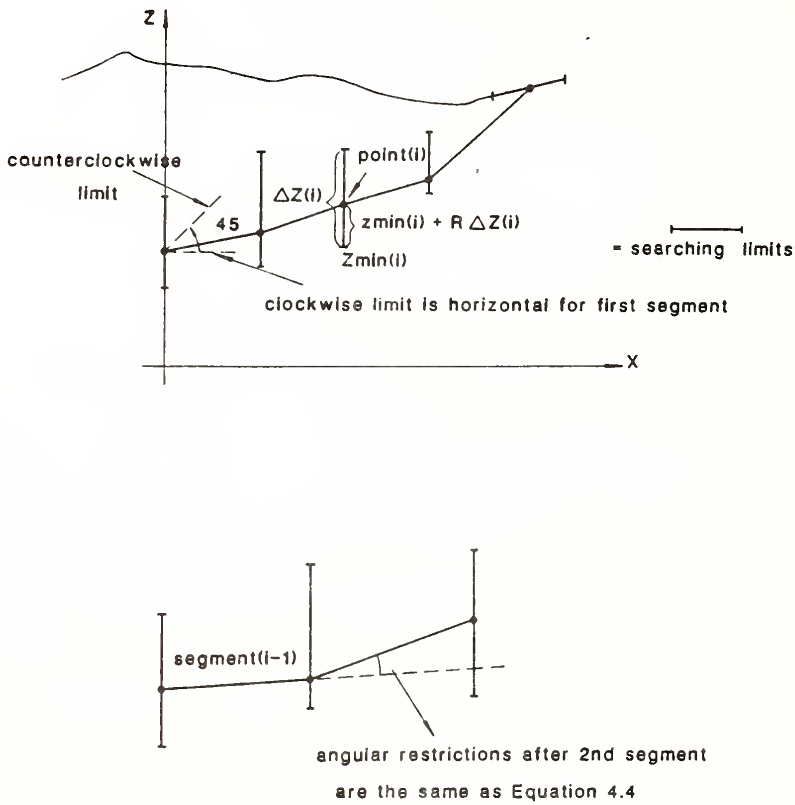


FIGURE IV.12 - General Aspects of the Surface Generation

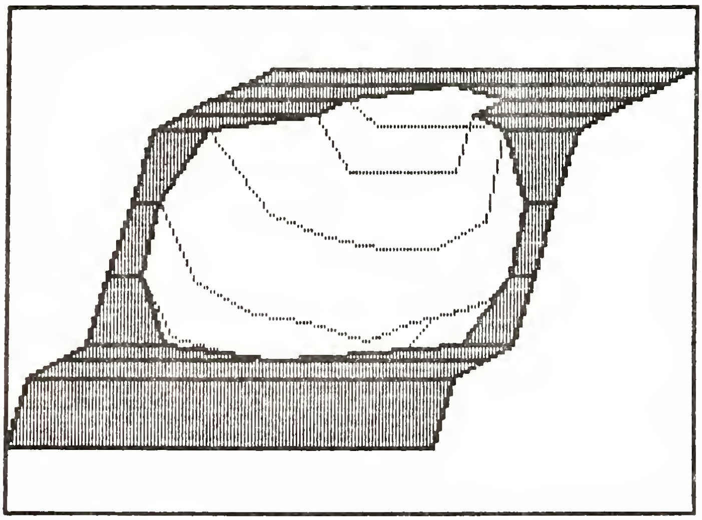


FIGURE IV.13 - First Example of Three Dimensional Surface

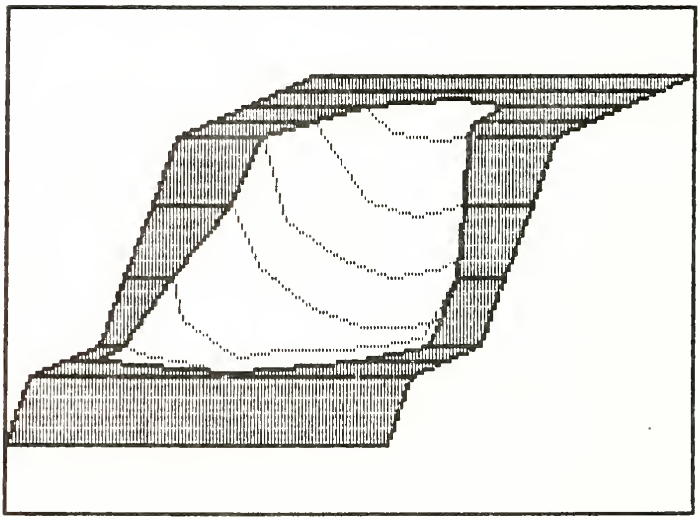


FIGURE IV.14 - Second Example of Three Dimensional Surface

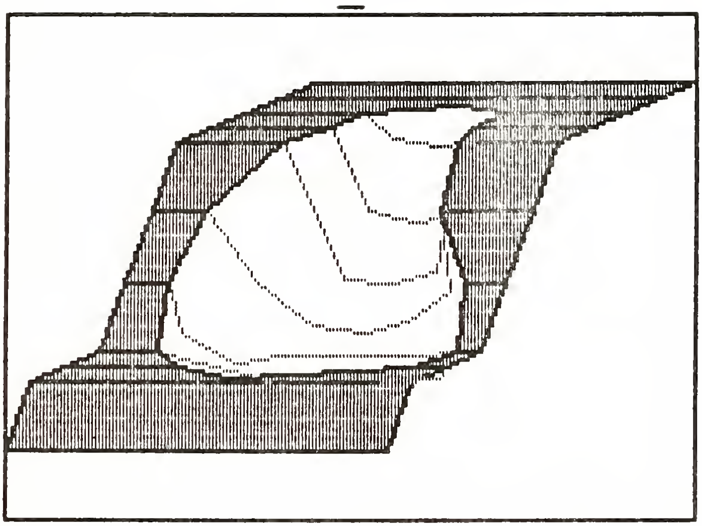


FIGURE IV.15 - Third Example of Three Dimensional Surface

V - DESCRIPTION OF THE METHOD OF ANALYSIS

V.1 - A General Method

The previous chapter presented the methodology for the generation of three dimensional surfaces . It became evident that the failure surfaces generated have very general nature and can cover a large variety of shapes. The surfaces will represent slides that can have either a translational or a rotational nature, and in order to calculate the factor of safety of both types, a general method of analysis had to be developed.

The method consists of an extension of the work presented by Chen (1981), and satisfies both force and moment equilibrium.

V.2 - Method of Columns

The analysis of the surfaces will be performed by dividing the soil mass in columns . This procedure expands the method of slices to three dimensional problems and, as mentioned in Chapter II, was introduced by Hovland (1977) and improved by Chen (1981) .

In the work developed here, the columns will be defined by the grid of points contained by the surface generated (FIGURE V.1). The columns will not necessarily have a square base . The columns of the boundaries of the surface can have trapezoidal or triangular bases (FIGURE V.2) . The use of the grid of points entered as data (for the generation of the surface) to define the columns was necessary to save memory (which was a major source of concern) and to speed the calculations.

V.3 - Assumptions

As it was shown by Chen (1981), the number of parameters included in a three dimensional analysis is much larger than for a two dimensional analysis. FIGURE V.3 shows a free body diagram of a column taken out of the sliding mass and TABLE 3.1 compares the number of parameters needed in typical two and three dimensional analysis.

In order to decrease the level of indetermination of the problem, assumptions were made to eliminate or fix the value of some unknowns. The assumptions made in this work follow the ones proposed by Chen (1981), and are listed as follows :

1) The failure mass is symmetrical about the main axis of sliding.

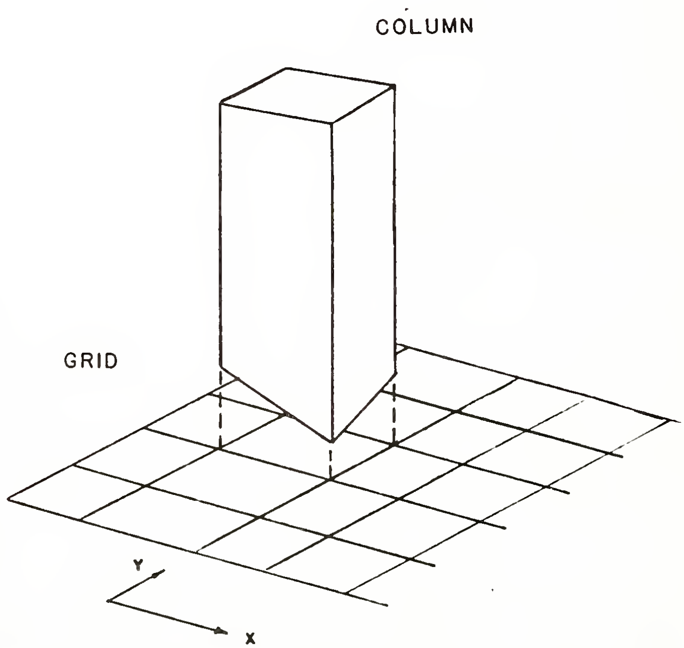


FIGURE V.1 - Columns are Defined by the Grid of Points

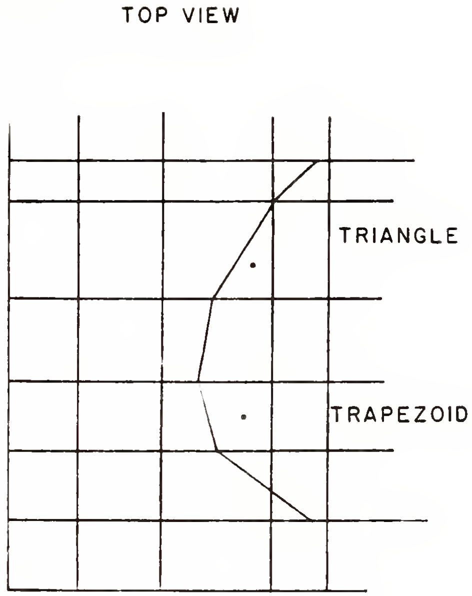


FIGURE V.2 - Triangular and Trapezoidal Bases

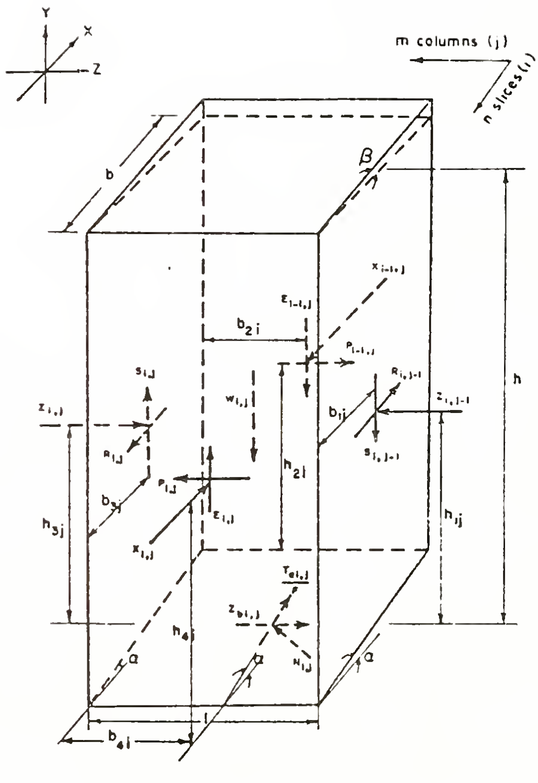


FIGURE V.3 - Free Body Diagram of a Column;
(After Chen, 1981)

TABLE V.1 - List of Unknowns in 2D and 3D cases; (After Chen, 1981)

Parameters	3-D Unknowns	2-D Unknowns
$Z_{i,j-1}$ $Z_{i,j}$	$(m+1)n$	0
$X_{i-1,j}$ $X_{i,j}$	$(n-1)m$	$n-1$
$R_{i,j-1}$ $R_{i,j}$	$(m-1)n$	0
$P_{i-1,j}$ $P_{i,j}$	$(n-1)m$	0
$S_{i,j-1}$ $S_{i,j}$	$(m-1)n$	0
$E_{i-1,j}$ $E_{i,j}$	$(n-1)m$	$n-1$
h_{1j} h_{3j}	$(m+1)n$	0
h_{2i} h_{4i}	$(n-1)m$	$n-1$
b_{1j} b_{3j}	$(m+1)n$	0
b_{2i} b_{4i}	$(n-1)m$	$n-1$
Z_{bij}	mn	0
N_{ij}	mn	n
F	1	1
R_{ext}	$2n$	0
S_{ext}	$2n$	0
	$12mn-5m+5n+1$	$5n-3$

2) Direction of movements along the "Y" direction only, and consequently the shear stresses along the "X" direction are zero.

3) The forces on the sides of the columns will be assumed as acting along the central vertical line of each side.

4) The intercolumn shear forces are assumed to be parallel to the bottom of the side of the column (FIGURE V.4) . The cohesive part of the shear force will act at a distance h/2 from the bottom and the frictional part of the shear force will act at the center of gravity of the normal stress distribution along the sides, which is considered linear between layers (FIGURE V.5) .

The inter-column shear forces at the two ends of each column are assumed to be a function of their positions, taking the largest values at the outmost points and decreasing to zero at the central section because of no relative movement in the middle section of the surface . The outmost shear forces, Rext and Sext an be obtained from equations 5.1, assuming that the Ko condition prevails:

$$\begin{aligned} Rext &= (0.5\ Ko\ \ h \tan \phi + c\ )\ b\ h \cos \\ Sext &= Rext \tan \\ R\ i,j &= Rext\ f(x) \qquad\qquad (\text{ Eq. } 5.1\ ) \\ hc &= h/2 \quad \text{and} \quad h = h/3 \end{aligned}$$

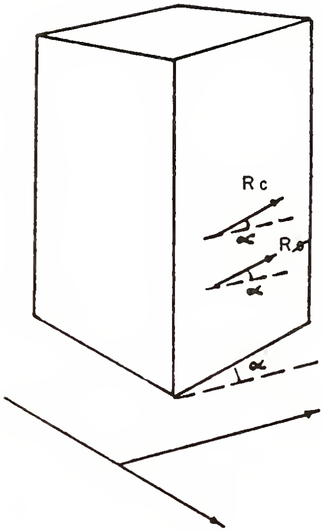


FIGURE V.4 - Intercolumn Shear Forces Parallel to the Bottom the Side

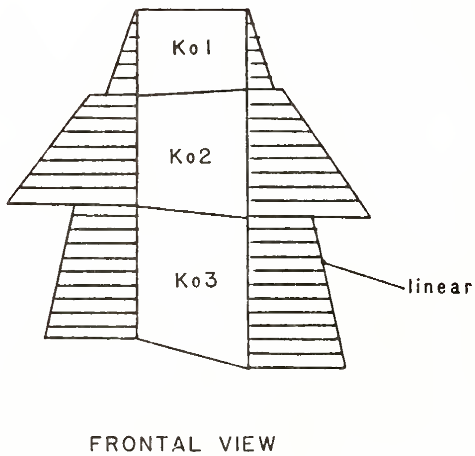


FIGURE V.5 - Linear Distribution of Horizontal Stresses Between Soil Layers

5) The interslice forces on the sides of the column are assumed to have the same inclination Teta throughout the whole failure mass.

V.4 - Derivation of Equations

The system of forces acting on every slice has to be in equilibrium and consequently, if we project and sum all forces in a local coordinate system with axes parallel ( ) and normal ( ) to the base of the column we get :

$$\sum F \quad = 0$$

$$N' (\tan \phi')/F + c'Ab /F - Q \cos(\alpha xy - \theta) - (W - Fv) \sin \alpha xy$$
$$+ R2 \cos(\alpha 2 - \alpha xy) - R1 \cos(\alpha xy - \alpha 1) - Fh \cos \alpha xy = 0$$

(Eq. 5.2)

$$\sum F \quad = 0$$

$$N' + u Ab + Q \sin(\alpha xy - \theta) - (W - Fv) \cos \quad xy + R2 \sin(\alpha 2- \quad xy)$$
$$+ R1 \sin (\alpha xy- \alpha 1) + Fh \sin \alpha xy = 0$$

(Eq. 5.3)

where

c′ = effective cohesion of the soil at the base of the column

ϕ' = shear angle of the soil at the base of the column

F = factor of safety

Ab = area of the column base

αxy = inclination of the central section of the base of the column with respect to the horizontal

R1, R2 = shear forces acting on the two ends of the column along the same " Y " cross section.

$\alpha 1$, $\alpha 2$ = inclinations of the intersection of the above two ends and the base.

Fv = earthquake vertical acceleration

Fh = earthquake horizontal acceleration

W = weight of column

From equation 5.3, we get :

$$N' = -u\ Ab - Q \sin(\alpha_{xy} - \theta) + (W - Fv) \cos \alpha xy - R2 \sin(\alpha 2 - \alpha xy) - R1 \sin(\alpha xy - \alpha 1) - Fh \sin \alpha xy$$

(Eq. 5.4)

Substituting equation (5.4) into equation (5.2) :

$$Q = \{ c'Ab/F - u\ Ab (\tan \phi')/F + (W - Fv) \cos \alpha xy ((\tan \phi')/F - \tan \alpha xy) + R2 \cos(\alpha 2 - \alpha xy) \{1 - (\tan \phi' \tan(\alpha 2 - \alpha xy)) / F \} - R1 \cos(\alpha xy - 1) \{ 1 + (\tan \phi' \tan(\alpha xy - 1))/F \} - Fh \cos \alpha xy (1 + (\tan \phi' \tan \alpha xy)/F \} / \{ \cos (\alpha xy - \theta) [1 + (\tan \phi' \tan (\alpha xy - \theta))/F] \}$$

(Eq. 5.5)

Now, to bring the system to equilibrium, the sum of all forces must be zero .

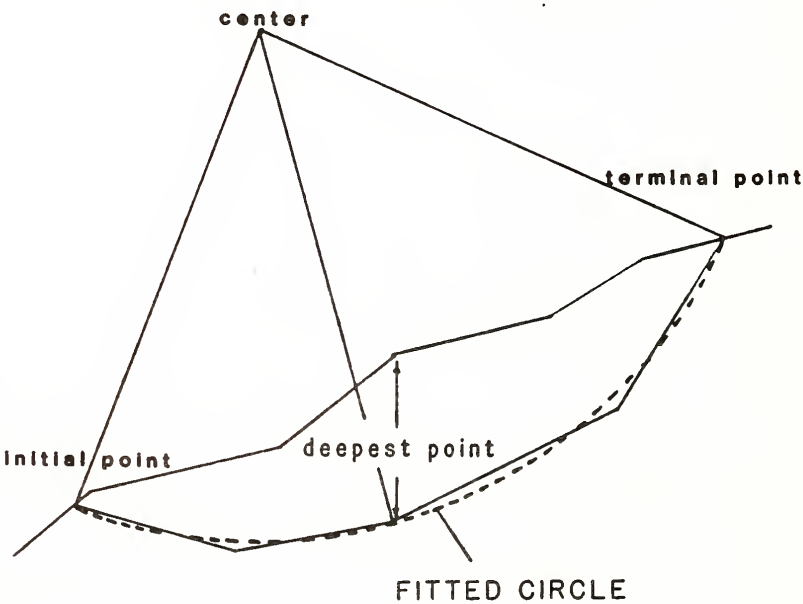
$$\sum Q = 0 \qquad \text{(Eq. 5.6)}$$

The sum of all moments about any point must also be zero. It is important to notice that, once the surfaces may assume any shape, there is not a fixed axis of rotation common to all of them . Even for each surface, there is no particular point about which to calculate the moment equilibrium . Thus, for each surface, the program will generate a point about which the moment equilibrium will be calculated.

The way the program generates this point is very simple. The program fits a circle that passes through three points of each surface, and the point about which the moment equilibrium is calculated, for that surface, is the center of that circle. The three points used are on the main sliding axis cross section . They are the initial point of the polygonal, the terminal point of the polygonal and the deepest point of the polygonal . The circle can be seen in FIGURE V.6 .

For that point we can write the momentum equilibrium as :

$$Q \cos(\theta - \alpha)\ (r - hq \cos\alpha\ ) = 0 \qquad \text{(Eq. 5.7)}$$



CROSS VIEW OF THE MAIN AXIS OF SLIDING

FIGURE V.6 - Adjustment of a Circle Through Three Points of a Surface

The same equation can be written as :

$$\{ Q\, r \cos(\theta - \alpha) - Q\, hq \cos \alpha \cos(\theta - \alpha) \} = 0 \qquad (\text{Eq. } 5.8)$$

On the above equation, the value of Q hq on each column can be calculated as a result of summing all the internal moments about the center of the base of that column . The resulting equation is:

$$Q \cos \theta \; hq + \sum_{i=1}^{nsoils1} (Rc1i\; dc1i + R\phi1i\; d\phi1i) \cos \alpha 1$$
$$- \sum_{i=1}^{nsoils2} (Rc2i\; dc2i + R\phi2i\; d\phi2i) \cos \alpha 2 - Fh\; dh = 0$$

(Eq. 5.9)

where:

nsoils1 = Number of soil layers on side 1

nsoils2 = Number of soil layers on side 2

Rc1i = Resulting cohesive force due to layer "i", on side 1 of the column

Rø1i = Resulting frictional force due to layer "i", on side 1 of the column

Rc2i = Resulting cohesive force due to layer "i", on side 2 of the column

Rø2i = Resulting cohesive force due to layer "i", on side 2 of the column

dc1i = Vertical distance between the point of application of cohesive force due to layer i on side 1 and the center of base of the column.

dø1i = Vertical distance between the point of application of frictional force due to layer i on side 1 and the center of the base of the column.

dc2i and dø2i = same of dc1i and dø1i but for side 2.

Fh = Horizontal earthquake force

dh = Vertical distance between the ceter of gravity of the column and the center of the base.

On the above equation the values of the forces and their positions of application can be obtained assuming the distribution of horizontal stresses linear with depth and conditioned by the Ko parameter of each soil. Based on the FIGURE we notice that :

a) The value of Rc1i is (ci B hi)/F where B is the thickness of

the column and hi is the height of the layer "i" at the middle section of side 1. The same is valid for Rc2i, on side 2.

b) The point of application of Rc1i is at a height hi/2 . The same is valid for side 2.

c) The value of $Ro\,\underset{(1 \text{ or } 2)}{\hat{}}\,i$, on any side of the column is given by:

$$Ro\underset{(1 \text{ or } 2)}{\hat{}} i = 0.5\ Koi\ [\ 2(\sum_{j=1}^{i-1} \gamma j\ hj\ ) + \gamma i\ hi\ ]\ hi \qquad (\text{ Eq. V.10 })$$

where j is the effective weight of the layer j.

d) The application point of the frictional force $Ro\,\underset{(1 \text{ or } 2)}{\hat{}}\,i$ is :

$$\text{height} = (L2 + 2\ L1)\ /\ 3(L1 + L2) \qquad (\text{ Eq. V.11 })$$

where:

$$L1 = (\sum_{j=1}^{i-1} \gamma j\ hj\ )\ Koi$$

$$(\text{ Eq. V.12 })$$

and $$L2 = (\sum_{j=1}^{i} \gamma j\ hj)\ Koi$$

The final system of equilibrium is composed of two non-linear equations, with only two unknowns : the factor of safety FS and the interslice force inclination theta . The system can, thus, be solved with successive iteractions, using a modified Newton's method.

VI - PREPARATION OF DATA

The main program will read data contained in a file specified by the user. This file can be created in two different ways. The first way is by using the routine " INDATA.BAS" supplied with the program. This routine interfaces with the user and requests the data needed for the analysis, adjusting the questions according to the answers of the user. This method of input is appropriate to the beginner and for avoiding format errors as well as missing data. The routine saves the input data in the file where it can be later accessed by both the graphical preprocessor and the main program.

The second way to create a data file is by using a standard word processor. Although more suceptible to errors, this is usualy faster than using the previously mentioned routine . Thus advanced users will probably prefer this option.

In order to simplify the generation of data files with word processors, all data used in the program are read in free format.

For the users who intend to create files with word processors, the required input data are outlined below. A new line should be started whenever a DATA CARD statement is encountered.

DATA CARD : Number of points in the "X" direction;
Number of points in the "Y" direction;

DATA CARD : Number of soils

DATA CARD : Coordinate x of points in the X direction (as many coordinates as fit in each line, until the last point)

DATA CARD : Coordinate y of points in the Y direction (as many coordinates as fit in each line, until the last point)

DATA CARD : Coordinate "Z" for all points in the Y direction, for a point X1 in the X direction. Repeat for each point in the X direction . Then repeat for each soil.

DATA CARD : Number of which point in the X direction will represent the main axis of sliding.

DATA CARD : For that main axis of sliding enter :
Y coordinate that limits the region where polygonals can start at that particular X .

Interval in the Y direction where surfaces can start.

Y coordinate that limits the region where polygonals can start at that particular X .

Interval in the Y direction where surfaces can terminate.

DATA CARD : For each point in the Y direction, starting with the second and terminating with the one before the last, enter :

X coordinate that limits the region where polygonals can start at that particular Y .

Interval in the X direction where surfaces are supposed to start .

X coordinate that limits the region where polygonals can terminate at that particular Y .

Interval in the X direction where surfaces are supposed to terminate.

DATA CARD : For each point in the X direction:

For all points in the Y direction at that particular X, enter:

Coordinate Z that limits the region where the surfaces can pass through.

Interval in the Z direction whre surfaces can pass through.

DATA CARD : For each soil enter:

Specific weight, cohesive intercept and shear friction angle, Ko .

DATA CARD : Unit weight of water.

DATA CARD : For each soil enter :

DATA CARD: Code for selection of water condition representation :

"r" if Ru parameter or "p" if piezometric line .

If "r" is selected then enter :

DATA CARD : Ru for the soil

If "p" is selected then :

DATA CARD : For each point in the X direction :

For all points in the Y direction at that particular X enter :

Coordinate Z of the piezometric surface at that point.

DATA CARD : Vertical earthquake acceleration

Horizontal earthquake acceleration

A listing of the Input routine, written in IBM BASIC, is presented in the Appendix B.

VII - GRAPHICAL PROCESSORS

In order to facilitate the checking of the geometry data , a graphical pre-processor was developed . This pre-processor displays all the geometry of the problem on the screen and allows the user to view three dimensional plots, cross sections of these three dimensional plots, or contour plots of the soil layers. It is important to notice that the screen resolution in the IBM-PC, when using the color mode, is not very high, and that the main purpose of the pre-processor is to help identify any major error resulting from mistyped data. Minor errors may not always be identified on the screen, and must be evaluated from the numerical printed output of the program.

The three dimensional view option (FIGURE VII.1) of the pre-processor allows the user to rotate the geometry about any axis, for any angle. The initial view, when no rotation at all is performed , is a view from the top of the geometry. The user is asked to especify rotation angles about the "X" axis, the "Y" axis and the "Z" axis , in this order. The user may check different combinations of angles without leaving the program .

Besides viewing the geometry three dimensionaly, the user can also compare the contour plots of the different soil layers

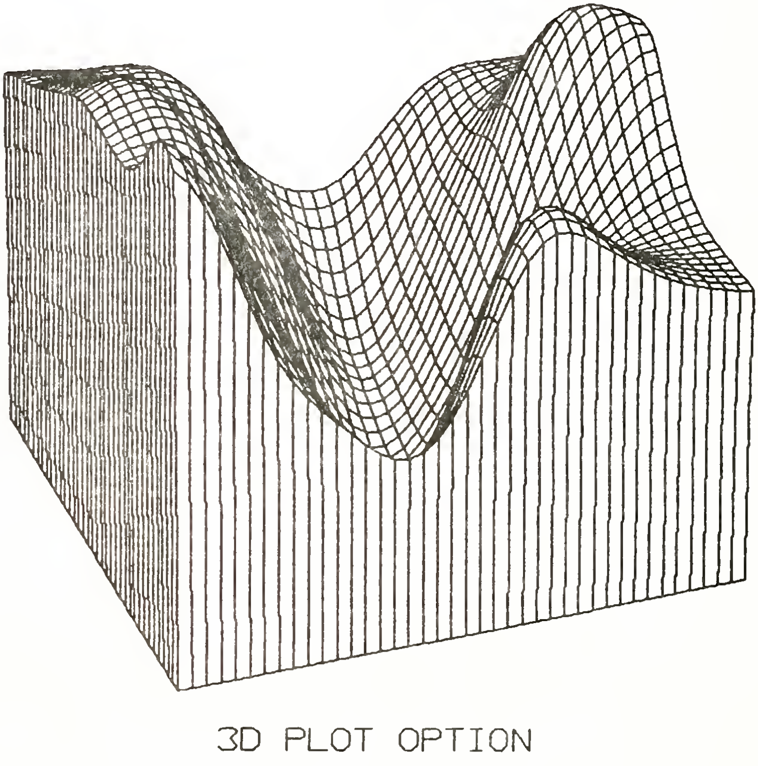


FIGURE VII.1 - 3D Plot Option

generated by the program to any topographical chart he might be working with. The contour option generates curves connecting points of equal height. The values of the heights are printed on every other curve (FIGURE VII.2). The interval of height between all curves is the same: one tenth of the maximum range in coordinate "Z" for the geometry to be plotted. The user can finally, check the data more accurately by plotting selected cross sections on the screen (FIGURE VII.3).

It is possible to dump the screen to the printer by using the "GRAPHICS" pack included in the IBM-PC operational system , and pressing the "SHIFT" and "PrtSc" keys at the same time, or by answering the program prompt about whether or not to print the generated graphics.

A typical session with the pre-processor is simulated below :

USER : Inserts disk in the drive and type "PREPRO". Press the return key.

PROGRAM : Displays " Enter name of data file name where geometry has been stored : ".

USER : Enter name and press the return key.

PROGRAM : Displays "Reading Data ... hold on "

```
PROGRAM : Displays " 1) 3D View and Cross-Sections"
                   " 2) Contour Plots             "
                   " 3) Exit the Program          "
                   "    Enter your option :       "
```

USER : Enter selected option.

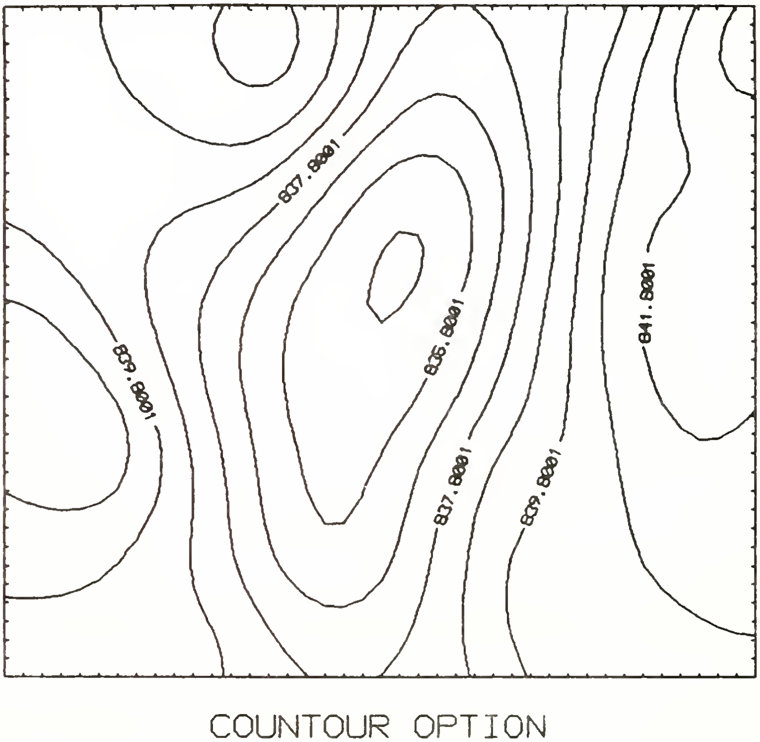


FIGURE VII.2 - Contour Option Plot

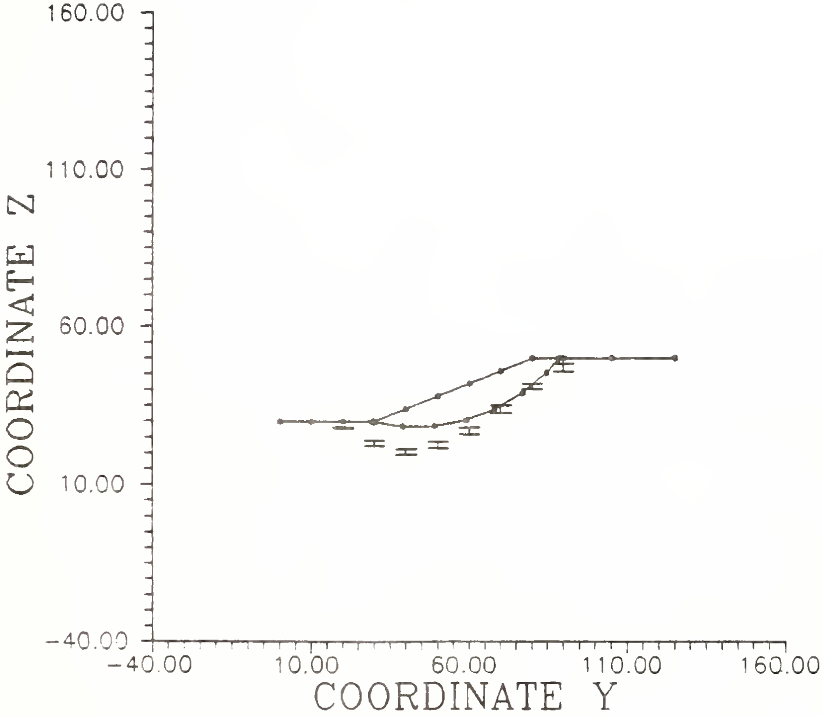


FIGURE VII.3 - Cross Section Option

IF SELECTION IS #1 THEN:

PROGRAM : Displays "Ready.
Enter angle of rotation about x axis :
Enter angle of rotation about y axis :
Enter angle of rotation about z axis : "

USER : Answers the prompts

PROGRAM : Opens a window in the screen and plot the geometry (FIGURE VII.1). If the geometry includes more than one layer, they are plotted one by one. As the soil layers are plotted according to the rotated coordinates, the layers farther from the user view point will be progressivelly hidden by the closer layers, as they are being plotted. The process can be held at any moment by pressing the keys "Ctrl" and "Num Lock" simultaneously. After all the geometry has been plotted, the routine stops .

USER : After checking the plot , press the "return" key.

PROGRAM : Displays "Enter cross section number to be displayed : "

USER : Enters the number of the cross section he wants to check. The cross sections are on the plane defined by the axes Z and Y. There are as many possible cross sections as the number of different coordinates X entered.

PROGRAM: Opens a window and plots the plane YZ at the number of the X coordinate selected by the user. The program automatically scales the axes so that the geometry is fully contained in the window.

USER : After checking the plot, presses the "return" key.

PROGRAM : Displays "Do you want to check another view ? (y/n)" .

USER : Answers the prompt. If the answer is yes, the section starts again, otherwise the three initial options are displayed again.

IF THE SELECTION IS #2 THEN:

PROGRAM : Displays "What soil # do you want to have plotted ?"
" (Enter 0 if you are finished) "

USER : Enter the number of the soil profile to be plotted.

PROGRAM : Displays "Do you want to print the contours ?"

USER : Answers Y for yes or N for no. If the answer is yes (Y), the user should turn on the printer at this moment.

PROGRAM : Will plot the contour plots of the soil profile on the left half of the screen and print the values of the contours on the right side. The program automatically finds the maximum and minimum coordinates of the select soil layer and generates 10 curve contours between these two values.
At the end, the screen content will be printed on an Epson compatible printer, automatically. To use the automatic printing capability, the user must load the IBM-DOS command "GRAPHICS" before running the program.

USER : After finishing with that particular layer, should press the return key.

PROGRAM : Displays "What soil # do you want to have plotted ?"
" (Enter 0 if you are finished) "

The program will repeat the operation until "0" is entered.

Then it will display the three initial options :

```
" 1) 3D View and Cross-Sections"
" 2) Contour Plots             "
" 3) Exit the Program          "
"    Enter your option :       "
```

And the process will start again.

VIII - RESULTS AND APPLICATIONS

In order to evaluate the methods proposed, a series of simple analyses were performed. A computer program, 3D-PCSTABL, was developed and used to find the most critical three dimensional surfaces and factors of safety of specific combinations of slope geometries and soil strengths. For comparison purposes, the program PCSTABL5 (Carpenter 1985), was used to locate the most critical two dimensional surfaces and their factors of safety. The method of slices of Spencer was used for this purpose. The results of these comparisons are presented in this chapter.

VIII.1 - The Parameters and Variables

The studies performed cover three major aspects:

a) Influence of the strength parameters in the ratio between three and two dimensional factors of safety.

b) Influence of the strength parameters and slope inclination in the agreement between the position of the three dimensional and the two dimensional most critical surfaces (checked on the main axis cross section).

c) Influence of the strength parameters and the slope inclination in the shape of the most critical surfaces.

d) Influence of pore water pressure

In all the cases, the soil is assumed to be homogeneous, and the searching boundaries were initially very general, being progressively refined until the critical surface was established. Different combinations of cohesion intercept and friction angle were tried, and the trends in the shapes of the most critical surfaces, the position agreement between the 3D and the 2D critical surfaces, and the ratio between the 3D and the 2D factors of safety were observed. The slope characteristics remained the same. The height of the slope is 6.1 m (20 ft), and the slope is variable, assuming inclinations of 1.5:1, 2.5:1 and 3.5:1. The density (unit weight) of the soil was considered to be 1930 kg/m3 (120 pcf). The strength parameters were taken as (1) $c'= 0$, $\phi'=40^{0}$; (2) $c'=14.4$ KPa (300 psf), $\phi'=25^{0}$; (3) $c'=28.7$ KPa (600 psf), $\phi'=15^{0}$. Cases were studied for two different water conditions: slope without water (ru = 0) and slope with ru = 0.5. These values were the same that Chen (1981) used in his work, for the sake of consistency.

VIII.2 - Results Obtained

VIII.2.1 - Influence of the Strength Parameters in the Ratio Between Three and Two Dimensional Factors of Safety.

The values for the critical factors of safety found by the two programs are presented in TABLE VIII.1, and the regression curves for the values of the ratios between the 3D and the 2D factors of safety are plotted in FIGURES VIII.1 and VIII.2.

TABLE VIII.1 - 3D and 2D Factors of Safety

Slope Angle	Case	ru = 0.0 Factors of Safety 2D	ru = 0.0 Factors of Safety 3D	ru = 0.5 Factors of Safety 2D	ru = 0.5 Factors of Safety 3D
	1	1.608	1.875	0.770	0.897
1.5:1	2	2.019	2.080	1.556	1.596
	3	2.439	2.456	2.165	2.188
	1	2.538	2.560	1.206	1.129
2.5:1	2	2.717	2.789	1.932	2.060
	3	3.245	3.339	2.587	2.965
	1	2.540	2.530	1.440	1.525
3.5:1	2	3.313	3.605	1.006	1.206
	3	3.157	4.181	2.691	3.639

1 - $c' = 0.0$ KPa $\phi' = 40^\circ$

2 - $c' = 14.4$ KPa $\phi' = 25^\circ$

3 - $c' = 28.7$ KPa $\phi' = 15^\circ$

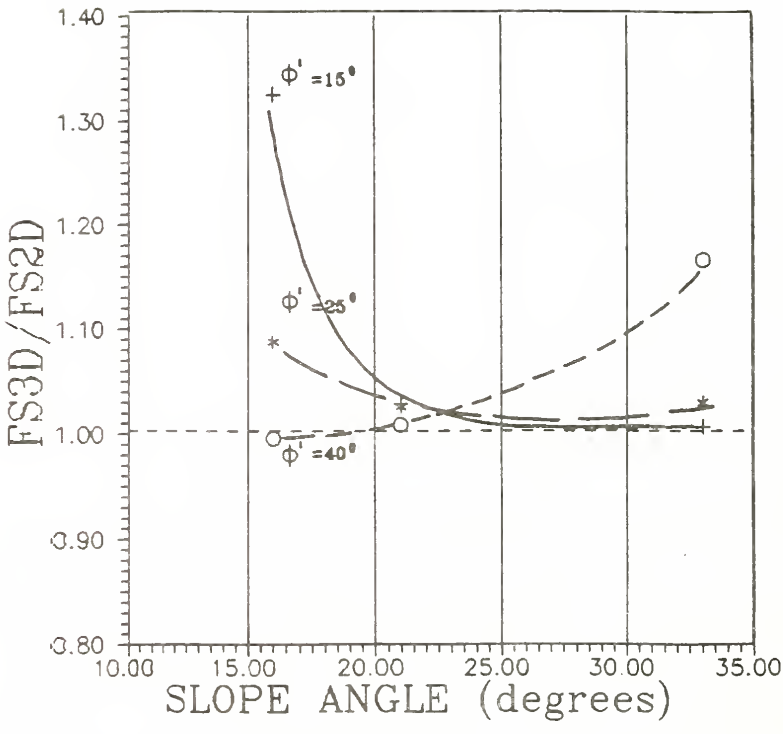


FIGURE VIII.1 - Ratios Between 3D and 2D Factors of Safety, ru = 0

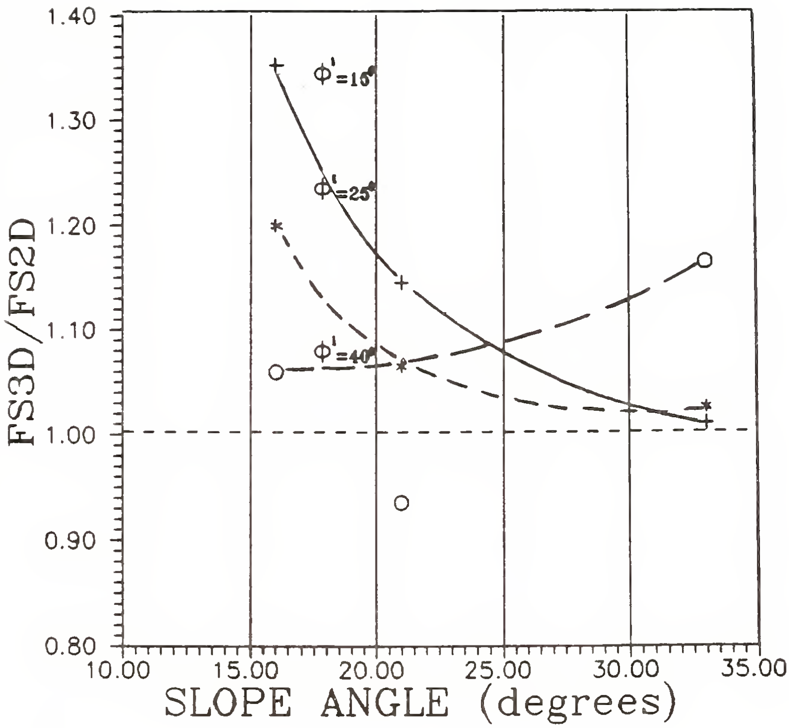


FIGURE VIII.2 - Ratios Between 3D and 2D Factors of Safety, ru =0.5

As we may notice, cohesive soils and cohesionless soils presented different behaviours. For highly cohesive soils (case 3), the ratio between 3D and 2D factors of safety decreases with the increasing inclination of the slope. This ratio can be as high as 130% for gentle slopes. This probably happens because, as it will be seen later, the volume of soil resisting the sliding movement (proportionally to the volume of soil driving the slide), is larger for gentle slopes than for steeper slopes.

For cohesionless soils (case 1), this behavior is the opposite, with the ratio increasing for steeper slopes. It was found that for steep slopes, for cohesionless soils, the three dimensional factor of safety can be less than the two dimensional. This result had also been found by Chen (1981).

Soils with significant cohesion and friction, will have an intermediate behavior that will depend on the relative importance of each strength component.

VIII.2.2 - Influence of the Strength Parameters and Slope Inclination in the Agreement Between the Position of the Three Dimensional and the Two Dimensional Most Critical Surfaces.

FIGURES VIII.3 to VIII.5 show the cross sections (on the main axis of sliding) of the ten most critical three dimensional surfaces plotted against the two dimensional surfaces found by PCSTABL5 for the slope with inclination 2.5:1, with Ru =0.0. FIGURES VIII.6 TO VIII.8 show the average depth of the same ten

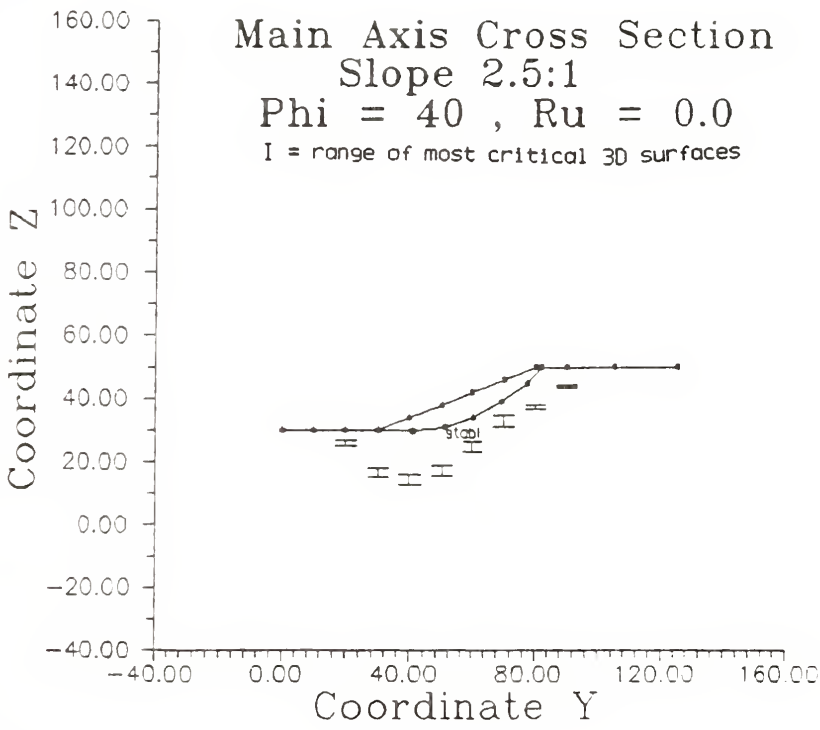


FIGURE VIII.3 - Agreement Between 3D and 2D Surfaces:
slope 2.5:1 , Case (1), ru = 0.0

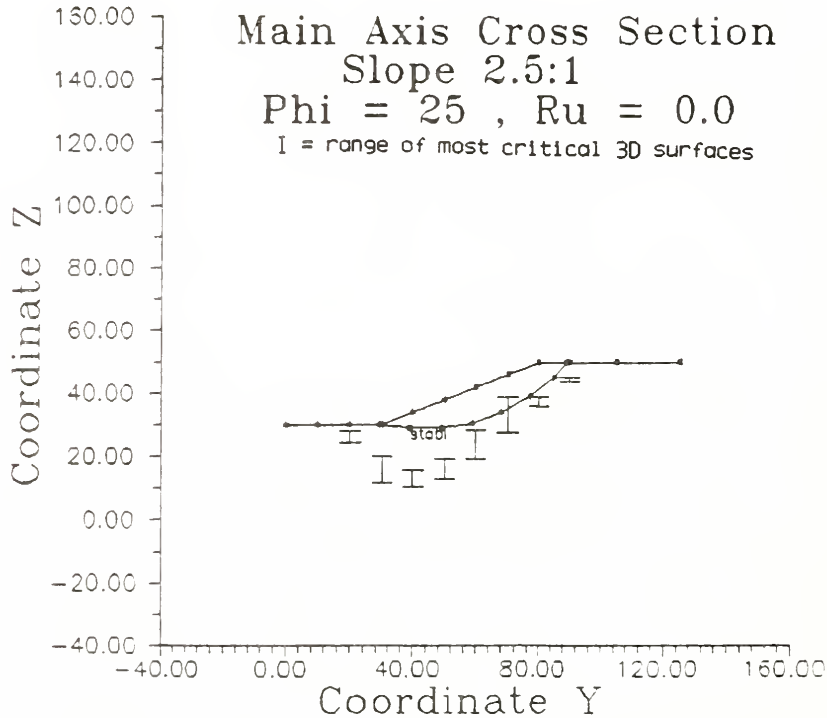


FIGURE VIII.4 - Agreement Between 3D and 2D Surfaces: slope 2.5:1 , Case (2), ru = 0.0

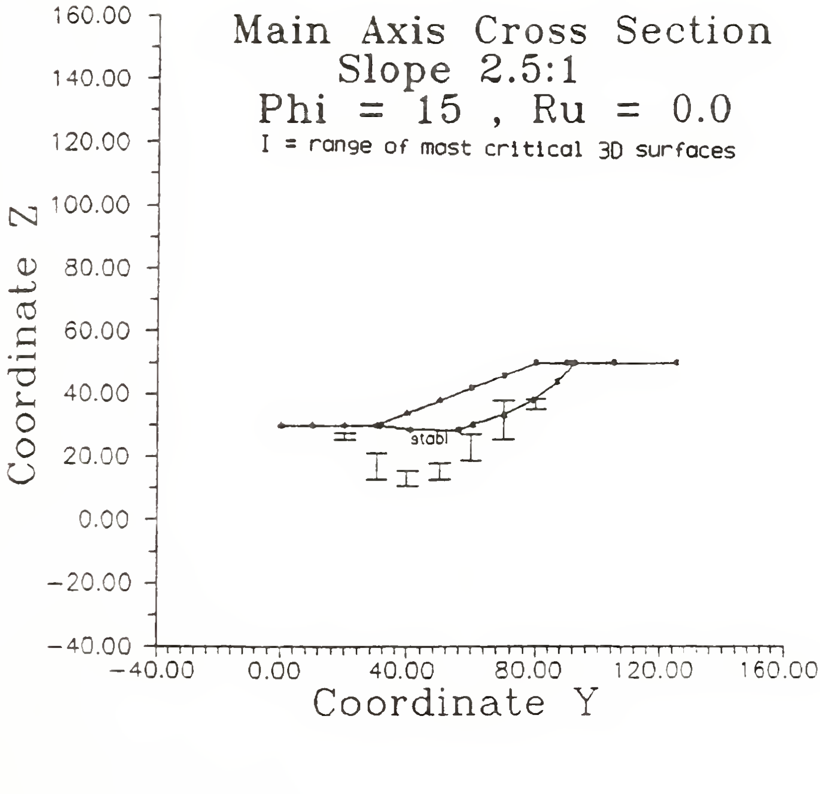


FIGURE VIII.5 - Agreement Between 3D and 2D Surfaces:
slope 2.5:1 , Case (3), ru = 0.0

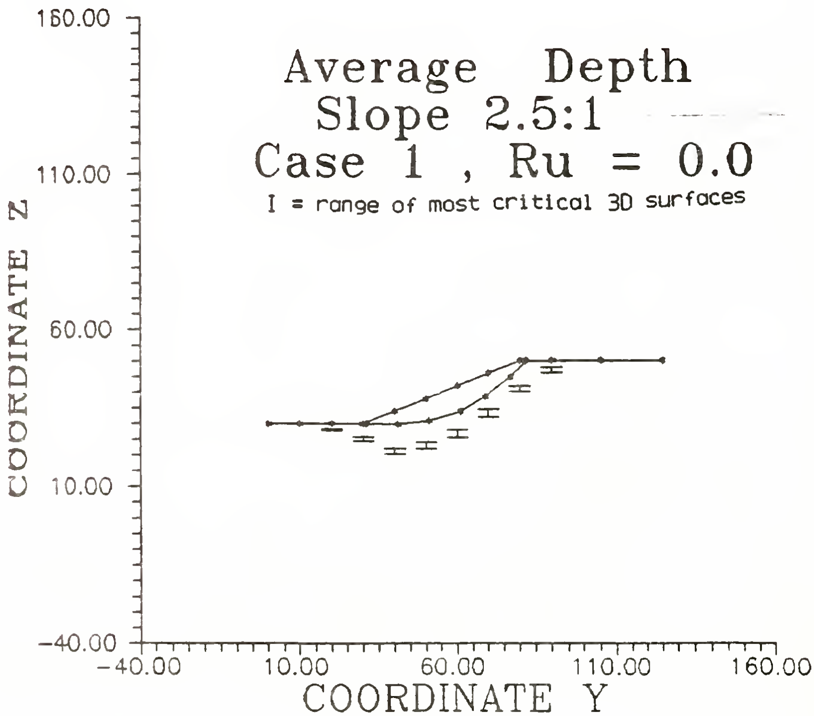


FIGURE VIII.6 - Average Depth of 3D and 2D Surfaces:
slope 2.5:1 , Case (1), ru = 0.0

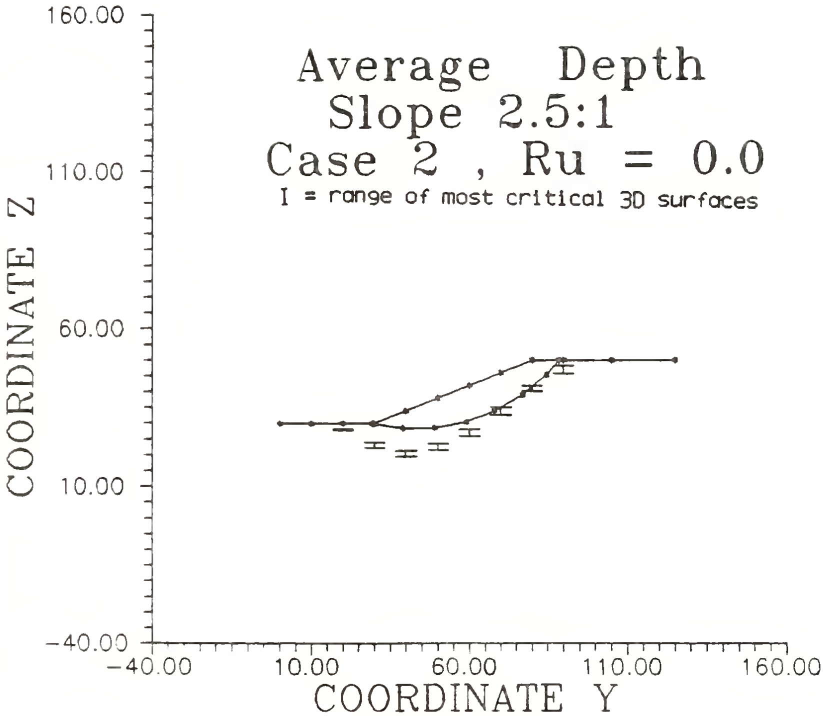


FIGURE VIII.7 - Average Depth of 3D and 2D Surfaces: slope 2.5:1 , Case (2), ru = 0.0

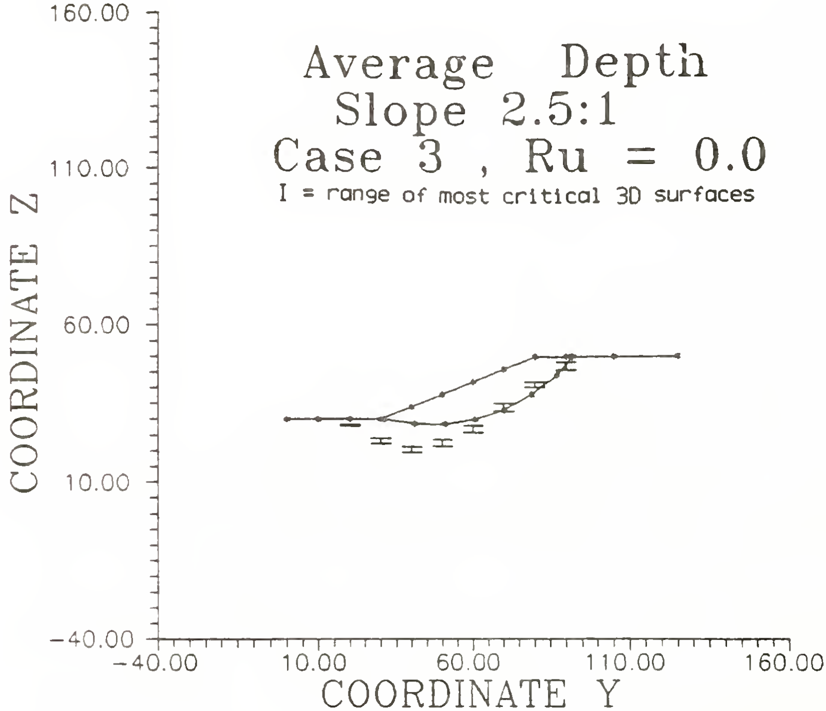


FIGURE VIII.8 - Average Depth of 3D and 2D Surfaces:
slope 2.5:1 , Case (3), ru = Ø.Ø

three dimensional surfaces, in comparison to PCSTABL5. The 3D surfaces are represented by vertical bars that show the intervals where the most critical surfaces are contained. It is interesting to notice that the scatter in the average depths is smaller than the scatter in the main axis cross sections. The average depths are more representative for comparison purposes, because they take into consideration the third dimension, which otherwise would not be represented in the cross section. The reader can refer to Appendix A to obtain the plot of all the results developed in this study. A close examination of these figures shows that for cohesionless soils, the three dimensional critical surface is deeper than the two dimensional one. The higher the cohesive component of the soil strength, the better the agreement between three and two dimensional surfaces.

It can also be noticed that the flatter the slope, for the same cohesion, the closer are the two and the three dimensional surfaces.

VIII.2.3 - Influence of the Strength Parameters and the Slope Inclination in the Shape of the Most Critical Surfaces.

Finally, FIGURES VIII.9 to VIII.11 show the effect of the strength parameters and slope inclination on the shape of the critical 3D surfaces, for case 2 . These figures show a topographic contour view of the slope with the critical surface found by the program. It is important to notice that the

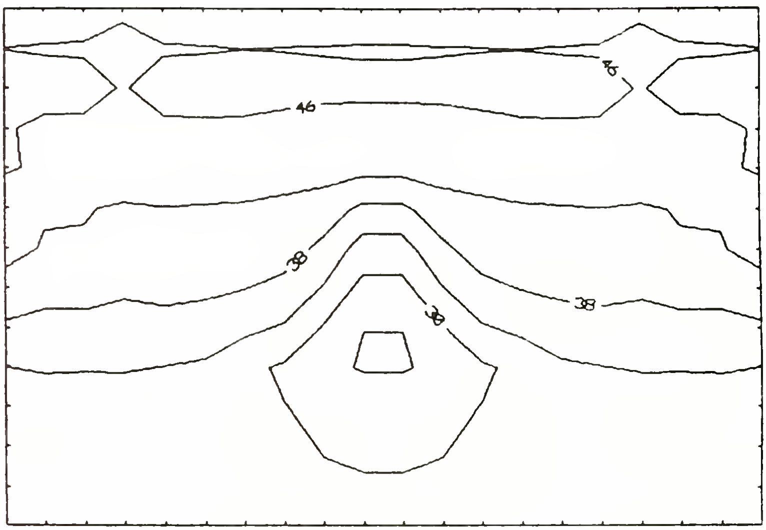


FIGURE VIII.9 - Contour Plot of 3D Surfaces:
slope 2.5:1 , Case (1), ru = 0.0

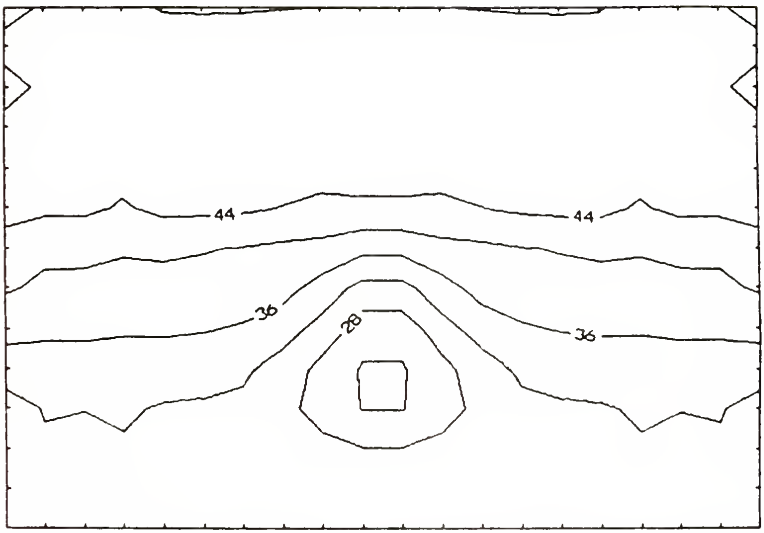


FIGURE VIII.10 - Contour Plot of 3D Surfaces:
slope 2.5:1 , Case (2), ru = 0.0

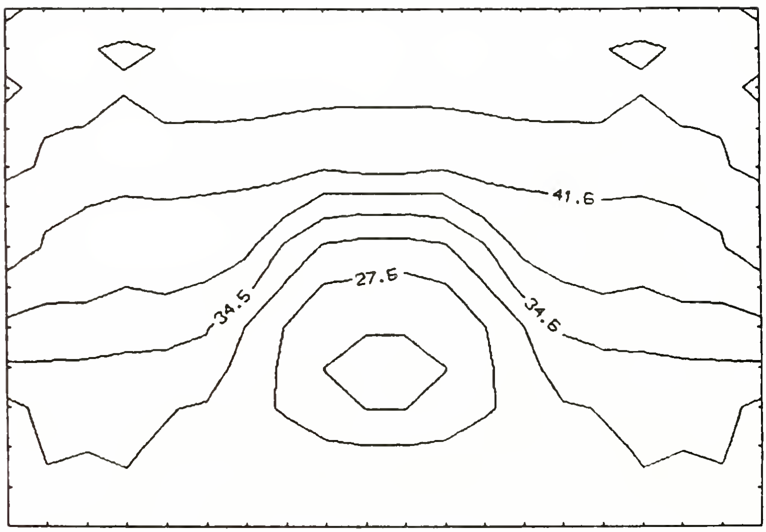


FIGURE VIII.11 - Contour Plot of 3D Surfaces:
slope 2.5:1 , Case (3), ru = 0.0

information in this figures are more qualitative, and that the accuracy of the figures are compromised by the interpolation process that calculates the elevations of the points. The complete set of figures can be found in Appendix A.

For cohesionless soils, the program found that the critical surfaces were narrower than for cohesive soils, and approximately block shaped. From the figures, no definite conclusion can be drawn about the influence of the slope inclination on the shape of the surfaces.

Cohesive soils, on the contrary, showed a tendency to have wider, ellipsoidal or spherical shaped surfaces, especially in steep slopes. As the slope flattens and the cohesion increases, the surface becomes wider. Apparently it is difficult for the program to define the exact shape of the critical surface, since the combination of shapes that would lead to essentially the same factor of safety is to high.

VIII.2.4 - Influence of Pore Water Pressure

In order to assess the effects of pore water pressure, all the former analyses were repeated using a pore pressure coefficient Ru of 0.5, and the resulting plots of the surfaces can be seen in Appendix A. The ratios between FS3D and FS2D are plotted in FIGURE VIII.2. It can be noticed that the pore pressure can increase the three dimensional effect. Besides the increase in the FS3D/FS2D ratio, the conclusions about the

agreement between 3D and 2D surfaces and the shape of 3D surfaces are the same as for Ru of 0.0.

IX - SUMMARY, CONCLUSIONS AND RECOMENDATIONS

A methodology for generating three dimensional failure surfaces with any shape, for slope stability analysis has been developed . The routine manipulates random numbers in order to create surfaces within specified regions of a given slope. Both symmetrical and non-symmetrical surfaces can be generated , even though no method is currently available to evaluate non-symetrical instabilities.

A general method of analysis , satisfying both moment and force equilibrium has also been developed. The method satisfies moment equilibrium about the center of a circle passing through three critical points of the surface being evaluated; the initial point, the terminal point and the deepest point of the surface.

The methodologies were implemented in a program (3D-PCSTABL) that runs in a IBM-PC microcomputer or compatible. To take advantage of the graphical capabilities of the micro, graphical processors were also developed , which allow the user to generate three dimensional plots of the geometry on the screen. These plots can be rotated in any direction , to improve the capacity of error detection in the input data. The basic hardware requirements are 256 Kb of memory , two floppy disks , a color card and a color monitor. The source code listing is given in Appendix B.

In order to check the performance of the program, a series of parametric studies were performed. Homogeneous slopes with different inclinations and strength parameters were analyzed using both the programs 3D-PCSTABL and PCSTABL5. From the results, one can conclude that landslides in cohesionless soils have a different three dimensional behaviour compared to slides in cohesive soils. The most relevant conclusions are:

1) For cohesive soils:

1.1 - The ratio between FS3D and FS2D is always greater than one. This ratio can be as large as 1.3 for gentle slopes, decreasing to almost 1.0 as the slope gets steeper. This is apparently related to the three dimensional shape of the most critical surfaces, which become wider at the foot of the slope and narrower at the top as the slope flattens.

1.2 - There is good agreement between the cross sections displaying the average depth of the most critical three dimensional surfaces and the two dimensional most critical surfaces. Apparently the agreement gets better as the cohesive intercept of the soil increases, and as the slope becomes flatter. For steep slopes, there was a tendency for the three dimensional critical surfaces to start beyond the toe of the slope.

1.3 - The three dimensional surfaces tended to have elliptical and spherical shapes. As the slope flattens, it becomes more difficult to establish the exact shape of the surface, since the surfaces become wider, and many surfaces lead to essentially the same factor of safety.

2) For cohesionless soils:

2.1 - The ratio between FS3D and FS2D can be less than one for gentle slopes, increasing as the slope's inclination increases. The maximum ratio was of the order of 1.15, for a slope of 1.5:1.

2.2 - The cross sections displaying the average depth of the most critical three dimensional surfaces are deeper than the two dimensional most critical surfaces. For steep slopes, just as for cohesive soils, there was a tendency for the three dimensional critical surfaces to start beyond the toe of the slope.

2.3 - The three dimensional surfaces tended to be approximately block shaped . There seems to be no definite relation between the shape of the surfaces and the inclination of the slope.

3) General Conclusions:

3.1 - The three dimensional shape of a landslide is related to the amount of shear strength and cohesion of the soils involved, as well as to the inclination of the slope.

3.2 - According to the values found for FS3D/FS2D, with slopes of 20° to 25°, two dimensional analysis should give satisfactory results (less than 5% difference from three dimensional analysis), for any combination of soils, as far as no pore water pressure is involved.

3.3 - Water influence was found to increase the three dimensional effect for all soils.

Even though the theory developed here is relatively sophisticated, there is a strong need for further development of the following topics:

1) Improve the generation of the three dimensional surfaces, by using methods such as the cubic spline interpolation to create smoother surfaces. This method could also be used to improve the geometric description.

2) Expand of the method of analysis for non-symmetrical surfaces and evaluate the errors involved in the assumption of symmetry.

3) Develop a better understanding of the distribution of intercolumn forces, and evaluate of the positioning of the thrust line. For this kind of study , more computer memory will be necessary.

4) Accomplish comparative studies between three dimensional analysis and actual failures, to access the accuracy of the new method developed here.

5) Since the surfaces are randomly generated , they constitute a Monte Carlo simulation analysis, and consequently , some kind of statistical projection of the final factor of safety can and should be performed.

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APPENDICES

The Appended Material (pp 108-275) is not included in this report. It may be obtained at cost by writing:

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